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A SIMULATION STUDY OF THE LEM DOCKING MANEUVER USING A MINIMUM IMPULSE CONTROL MODE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
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A SIMULATION STUDY OF THE LEM DOCKING MANEUVER USING A MINIMUM IMPULSE CONTROL MODE

SUMMARY

This document states the objectives, approach, and findings of a LEM docking simulation study completed at NASA-MSC. The purpose of the study was to test an open loop mode of achieving fine control of LEM attitude and translation near dock. The mode employed fixed-impulse thrusters actuated single pulse per control stick throw. One hundred pound thrusters were assumed.

Results of the study show that the LEM can be docked satisfactorily using the described control mode. In addition, a qualitative evaluation of the desired magnitude of individual pulses is presented. The results show the range of impulse levels best suited to the mode.

INTRODUCTION

Reaction control thrusters having outputs of 100 pounds or more have been found desirable for several phases of the LEM mission (refs. 1 and 2). These phases include descent and landing (where LEM moments of inertia are relatively large), rendezvous, and the early portion of docking. During the later portion of docking, however, where comparatively fine control of LEM attitude is required, a small thruster size (5 to 10 lbs.) is preferable (ref. 3). The question arises as to how a single thruster size, say 100 pounds, can be adapted to all phases of the LEM mission.

Adaptation of the 100-pound thruster to fine control of LEM docking can be accomplished without great difficulty by means of feedback control loops such as the "rate command" or "attitude hold" control loops described in references 1 and 3. But adaptation of the 100-pound thruster to open loop control of LEM docking (desired as a backup control mode) poses a more serious problem. It was found (refs. 1 and 3) that open loop control of the LEM in docking was both imprecise and dangerous if 100-pound thrusters, actuated ON/OFF, were employed. The purpose of the simulation study reported here was to determine whether or not the 100-pound thruster, actuated in pre-established impulses of relatively small magnitude, could provide sufficiently fine control of the LEM for satisfactory open loop docking.

The fixed-impulse mode of thruster actuation, frequently called a minimum impulse mode, yields a single impulse of fixed magnitude for any given ON command. (This is accomplished by pre-establishing thruster ON time.) To get another impulse, it is necessary to reeenter the controller (bring it within its deadband) and then apply it again.

SYMBOLS

SYMBOL	UNIT	DEFINITION
Fx, Fy, Fz	lb	Total translational thrust in x, y, and z
ΔF_{ϕ}, ΔF_{ϕ}, ΔF_{ψ}	lb	Translation thrust due to unbalanced attitude control thrust
D_{ϕ}	ft	Thruster moment arm about cg in roll
D_{θ}	ft	Thruster moment arm about cg in pitch
D_{ψ}	ft	Thruster moment arm about cg in yaw
I_{xx}	slug-ft^2	Principal moment of inertia, x-axis
I_{yy}	slug-ft^2	Principal moment of inertia, y-axis
I_{zz}	slug-ft^2	Principal moment of inertia, z-axis
K	rad/sec	Stick sensitivity
Lc	ft-lbs	Attitude control moment, roll-axis
Mc	ft-lbs	Attitude control moment, pitch-axis
Nc	ft-lbs	Attitude control moment, yaw-axis
M	slugs	Mass
p	rad/sec	LEM angular rate about x-axis
q	rad/sec	LEM angular rate about y-axis
r	rad/sec	LEM angular rate about z-axis
R	ft	Range, that is, line of sight distance to the command module, CM

SYMBOL	UNIT	DEFINITION
$\dot{R}$	ft/sec	Range rate or closing rate
u,v,w	ft	Translational rates in the LEM body axis system
x,y,z	ft	Displacement of the LEM with respect to the CM
Δx, Δy, Δz	ft	cg offset from nominal
β	rad	Error signal for attitude thruster
ϕ	rad	Euler angle in roll
Θ	rad	Euler angle in pitch
ψ	rad	Euler angle in yaw
$\tilde{\psi}$	rad	Azimuth of line of sight to CM
$\tilde{\Theta}$	rad	Elevation of line of sight to CM

DESCRIPTION OF THE DOCKING SIMULATION

General.- The flight mechanics of the LEM in near approach to the Apollo spacecraft (command module, CM) were programed on a general purpose analog computer (EA231R). A functional mock-up of the LEM cockpit was used to provide pilot control inputs to the computer and computer output displays to the pilot. A further visual cue to the pilot was an oscilloscope mounted at the pilot's eye level. (See fig. 1a.) It was intended to simulate a window of the LEM from which the target, the CM could be viewed. The CM was represented by the cathode ray dot on the oscilloscope. The pilot was enabled by these displays to control the simulated LEM according to the docking situation portrayed, thus, in effect, closing a control loop around the LEM.

Displays.- The instrument displays, which were mounted in the simulated LEM cockpit, are shown in figure 1b. These instruments were used to present the following flight information to the pilot:

(1) The five-inch diameter three-axis "eight-ball" displayed LEM pitch, yaw, and roll attitudes. The cross bars of this instrument displayed the azimuth and elevation of the pilot's

line-of-sight to the CM; the side and bottom meters of the instrument showed body angular rates, q and r.

(2) The two vertical-scale meters displayed range and range rate.

The oscilloscope visual cue, described briefly above, was driven by signals proportional to the azimuth and elevation of the pilot's line-of-sight to the CM. The pilot's eyes were assumed to be at the LEM center of gravity. The oscilloscope represented a window located 12 inches in front of the pilot's face subtending a 20° right circular cone with apex centered at the pilot's eye.

Controllers.- The vehicle controllers were a three-axis attitude controller operated by the pilot's right hand and a three-axis translation controller operated by the pilot's left hand. They are shown in figures 1a, 1c, and 1d. The attitude controller commanded pitching motion when moved backward or forward in the pitch plane, roll motion when moved from side to side, and yawing motion when the knob was twisted to right or left with the fingers. The translation controller commanded forward or rearward translational when pushed in or pulled out, sideward translation when moved to right or left, and vertical translation when lifted or depressed.

Control system.- The attitude control system was operated open loop, that is, it employed neither rate nor attitude feedback. The thrusters were used in a minimum impulse mode (fig. 2), actuated single pulse per control stick throw. An ON-OFF system (fig. 3) was also used for attitude control in a dual mode to be described below. The translation control system consisted of the analog solution of equations 3 in the appendix. Translation thrust could be actuated ON-OFF or SPPT (single pulse per throw).

A dual mode of attitude control was tested on the attitude controller. In the dual mode, fixed-impulse SPPT thruster control (fig. 2) was combined at short control stick throw with conventional ON/OFF thruster control (fig. 3) at large control stick throw. (The purpose of the combination was to provide fine attitude control of the LEM in docking with a coarse control option.) The fixed-impulse mode was actuated at 5 percent of total stick throw, the ON/OFF mode at 90 percent.

Assumptions.- The following assumptions were made in simulating the docking operation:

(1) Small angle approximations in the docking equations of motion were used. (See Appendix.)

(2) The sequencing of individual thrusters, the synchronization of thruster pairs, and the misalinements in thruster mounting were

regarded as incidental to the central question of the study. They were, therefore, not simulated. One-hundred-pound thrusters were assumed for all runs.

(3) Thruster impulse shape was assumed rectangular. (It is misleading, therefore, to associate the time base of the analog impulse with the time base of the nonrectangular impulse of an actual thruster.)

(4) Electro mechanical transport lag in thrust application (about 6 milliseconds) was assumed to have negligible effect upon the handling qualities of the LEM, hence was not simulated.

(5) The effects of orbital mechanics on the final stages of docking were assumed negligible, hence were not simulated.

VEHICLE CONFIGURATION

The basic configuration of the LEM (as to thruster location, docking weight, axis-definition, et cetera) had not been specified at the time the subject simulation study was made. Therefore, a preliminary configuration (ref. 4) was used. This configuration, which may be called the light configuration, is represented in figure 4. Its mass, moments of inertia, and control moments are listed below:

$$M = 122 \text{ slugs}$$

$$I_{xx} = 560 \text{ slug-ft}^2$$

$$I_{yy} = 1760 \text{ slug-ft}^2$$

$$I_{zz} = 1760 \text{ slug-ft}^2$$

$$L_c = 1000 \text{ ft-lbs}$$

$$M_c = 1200 \text{ ft-lbs}$$

$$N_c = 1200 \text{ ft-lbs}$$

Some data runs were taken with a later configuration of the LEM (ref. 4). This configuration, called the heavy configuration, is shown in figure 5. Its mass, moments of inertia, and control moments were:

$$M = 850 \text{ slugs}$$

$$I_{xx} = 1200 \text{ slug-ft}^2$$

$$I_{yy} = 2500 \text{ slug-ft}^2$$

$$I_{zz} = 2500 \text{ slug-ft}^2$$

$$L_c = 800 \text{ ft-lbs}$$

$$M_c = 800 \text{ ft-lbs}$$

$$N_c = 800 \text{ ft-lbs}$$

TEST PROGRAM

General.- One-hundred-ten test runs were made. Seventy-seven of these were made with R. Davidson as pilot. J. Brickel was pilot for nineteen runs, and E. White for fourteen. (Messrs. Davidson and Brickel are pilots of long experience, and Capt. White is an astronaut.) The runs were made to obtain pilot ratings of LEM handling quality for selected impulse levels. Most of the runs were made with varying degrees of cross-coupling.

The runs were begun with the simulated LEM relatively close to dock (50 ft.), because, as indicated in the introductory remarks, the study was concerned with close-in docking.

Initial conditions.- Early in the process of making test runs, it was decided to zero all initial conditions, except range and range rate. The reason for this step was that the pilots were first nulling the imposed LEM rates and attitudes and then performing the docking task. They were thus dividing the test run into two separate tasks which really called for separate ratings.

A LEM to CM separation of 50 feet and a closing rate of 1 ft/sec were the non-zero initial conditions of the test runs.

Procedure.- Each simulated docking run required the pilot to bring the simulated LEM to a docked condition from a point fifty feet out. The docked condition was defined as a mating of the contact surfaces within prescribed velocity, displacement, and interface angle limits. These limits, or performance goals, are listed in table I. The pilot was required to rate LEM attitude and translation handling qualities for each run according to the Cooper Scale, table II.

Parameters varied.- Test runs were made using a different impulse level for each run. Impulse levels per thruster were varied from low (0.2 lb sec), practically unattainable values to values (4 lb sec) providing control authority on the order of that produced by the pilot blipping the controller. Center-of-gravity offset and thrust differential between thruster pairs (asymmetry) were also varied. The effect of these deviations is to produce LEM rotation where translation is commanded, or LEM translation where rotation is commanded. Center-of-gravity offset was increased up to 2 feet per axis, and thrust asymmetry up to 20 percent per thruster pair.

PRESENTATION OF TEST RESULTS

Figures 6 to 10b inclusive show pilot ratings of LEM docking control quality versus thruster impulse level. Figures 6a, 6b, and 6c permit comparison of three pilots' ratings of the docking task under a single adverse condition, that is, a cg offset along the vertical axis of the LEM. Figures 8a and 8b provide a means by which comparison can be made of two pilots' ratings of the docking task with (1) cg offsets in each of the three LEM axes, and (2) thrust asymmetries in the three sets of thruster pairs forming the three attitude couples. Figures 10a and 10b yield the same comparison where cg offsets are 2 feet per LEM axis and thrust asymmetries are 20 percent per attitude couple. Figures 7 and 9 give ratings of docking control assuming the heavy LEM configuration with thrusters mounted as shown in figure 5.

The curves of figure 9 flatten out in a range of impulse levels from about 2.5 to 4 lb sec per thruster. This flatness cannot persist, however, since all pilot rating plots bend upward as impulse values are increased toward levels representative of ON/OFF operation.

The attitude angles, angular rates, accelerations, translations, and translational rates of the LEM with respect to the SCM (the latter regarded as fixed) were recorded continuously. So were attitude and translation propellant consumption. The end conditions, values of the remarked quantities at dock, while appearing on strip charts, were also tabulated from digital voltmeter readings of the analog computer outputs. Their distribution is plotted in the bar graphs of figures 11 and 12, and their averages in the curves of figures 13 to 19, inclusive.

DISCUSSION OF RESULTS

The Fixed-Impulse mode.- The most important single finding of this study is that the LEM, equipped with large thrusters and operating without benefit of rate or attitude feedback, can be docked satisfactorily. That the problem of docking open loop with large thrusters can be solved by employing a fixed-impulse, SPPT mode of thrust application is evident from the graphs of this report. Pilot ratings of LEM attitude and translation control quality with the fixed-impulse mode applied (figs. 6-10) show a degree of control acceptable in a back-up control mode. From the cited figures and the Cooper Scale (table II), it appears that acceptable control is possible over a fair range of impulse levels. Relatively good control is seen to be possible for impulse levels ranging from 1.5 to 2.5 lb sec. (Impulse levels of this range are attainable on actual 100-lb hypergolic thrusters.)

The fixed impulse, SPPT mode of thruster control, by enabling fine control of the LEM in docking, permits the docking task to be completed without undue stress on the docking fixtures. In order that stress limits may not be exceeded, maximum allowable terminal docking conditions must be specified. The terminal conditions specified for this study were the suggested performance goals of table I. It can be seen from the bar graphs of figures 11 and 12, and the average curves of figures 13 through 16, that the contact or end conditions of the simulated docking runs met the suggested performance goals very well. Nevertheless from figures 11 and 13 it is evident that terminal angular rates were larger than 1 deg/sec in some instances, particularly in roll. It must be remembered, however, that the visual motion cues of the simulator were limited. Had a three dimensional view of the command module been available, a closer pilot estimate of final angular rates and attitudes would probably have been possible. (Terminal roll rates were higher than terminal pitch or yaw rates because the display instrumentation used was considerably less sensitive in its indication of the former than the latter.)

Use of the fixed-impulse, SPPT mode of thruster actuation for open loop control of the LEM in docking, requires skill on the part of the pilot. The pilots flying the test runs for this study acquired this skill quite rapidly.

Fuel utilization.- The fuel usage of the fixed-impulse mode was not compared in this study with the fuel usage of other modes. There is, however, no reason to suppose that it would be greater. The plots of fuel consumption (averages of all runs) versus impulse level show that most fuel is used in translation. (Compare figs. 17 and 18.) Average attitude fuel usage (fig. 19) increases almost linearly with impulse size, probably because overshoot increases linearly with impulse size.

Cross coupling effects.- The effects of thrust differential between members of a thruster pair (thrust asymmetry) and center of gravity offset do not appear to pose serious LEM handling problems for the pilot for asymmetries and offsets well in excess of realistic values. Thrust asymmetry alone, it was observed by pilot comparison of runs with and without thrust asymmetry (20 percent), had negligible effect on LEM control. Indeed, pilot ratings of the heavily cross-coupled runs including both thrust asymmetry and cg offset (figs. 8a, 8b, 10a, and 10b) were without doubt primarily ratings of cg offset effects rather than thrust asymmetry effects. The reason for the insignificance of 10- to 20-percent thrust asymmetry in the fixed-impulse mode is that thrust in this mode, especially for the smaller impulse levels, is applied for a very short time. Thrust asymmetry effects can thus be corrected before they become large. In the ON/OFF mode on the other hand, thrust asymmetry effects become comparatively large even in the short time duration of the fastest pilot-blipped impulse.

On the assumption, then, that pilot ratings of the heavily cross-coupled runs are primarily ratings of the effects of cg offset, there is apparent disagreement as to the seriousness of the effects of cg offset on the quality of LEM handling. (Compare figs. 10a and 10b.) This seems, however, to be a matter of individual pilot tolerance. In any event, large cg offsets should certainly cause greater LEM handling difficulty than small offsets cause. Support for this assertion can be gained from a comparison of figures 8a and 10a. A further conclusion about cg offset which might be drawn from comparison of figures 6a and 8a, 6c and 8b is that a center-of-gravity offset having components along all three axes appears to cause the pilot little if any more maneuvering difficulty than a center-of-gravity offset along one axis only.

The dual mode.- It was noted earlier that when high initial rates were imposed on the LEM, the pilot first nulled those rates and then applied himself to the docking task. The conclusion to be drawn from such action is that fine control of the LEM is not likely to be called for until the coarse control task has been completed. Moreover, once the high initial rates requiring coarse control have been nulled, there is ordinarily no need to return to coarse control. Barring malfunction, the potential source of undesired rates is now no more powerful than the means available to null those rates, that is, the authority of the fine control system itself.

Bearing the above comments in mind, it would seem unnecessary to have both a fine and coarse control capability simultaneously available on a controller. Moreover, the simultaneous presence of both capabilities on a given controller seems undesirable in that it admits the danger of applying high rates inadvertently. If it should be found desirable to combine an ON/OFF thruster control mode with a fixed-impulse mode on the same controller, a more positive mode isolation than stick throw distance, for example, a stick force gradient, would appear essential to safe use of the dual mode.

Because the pilots first solved the problem of nulling high rates before turning to the problem of docking, their ratings of docking control quality using the dual mode could hardly be different from their ratings of that quality using the fine control mode. Hence, no ratings of the dual mode appear in this report. A proper evaluation of the dual mode would require test runs involving the introduction of sudden malfunctions.

Impulse levels suitable for fine control of the LEM are not in general of sufficient magnitude for the tasks of coarse control. They may, therefore, be unequal to the task of nulling the high rates induced by a malfunction, or even the rates residual to rendezvous. A readily available coarse control capability in the event of malfunction is clearly necessary.

CONCLUSIONS

(1) The fixed-impulse, single-pulse-per-throw mode of thrust application can be used open loop for fine control of the LEM in docking, even in the presence of large cross-coupling.

(2) Use of the mode requires skill on the part of the pilot, which, however, is acquired rapidly.

(3) The impulse level best suited to fine control of the LEM in docking will fall in the range of 1.5 to 2.5 lb sec per thruster. These impulse levels are not of sufficient magnitude to accomplish the tasks of coarse control.

(4) A thrust asymmetry of 20 percent per thruster pair in the fixed impulse SPPT mode poses by itself no significant handling problems for the pilot.

(5) A center-of-gravity offset of 2 feet per axis causes the pilot appreciably more difficulty in maneuvering the LEM than a center-of-gravity offset of 1 foot per axis.

(6) A center-of-gravity offset having components along several axes of the LEM appears to cause the pilot little more difficulty in this mode than an offset of the same magnitude along one.

REFERENCES

1. Hackler, C., Guthrie, G., and Moore, T.: "Preliminary Study of the Pilot-Controlled LEM Docking Maneuver." NASA Project Apollo Working Paper No. 1075.

2. Cheatham, D. and Moore, T.: "Study of the Attitude Control Handling Qualities of the LEM During the Final Approach to Lunar Landing." NASA Project Apollo Working Paper No. 1074.

3. Willman, J.: "Lunar Docking Simulation Study." North American Aviation Report, NA 63H-82. February 1963.

4. Russo, J.: Grumman Aircraft Engineering Corporation Presimulation Report No. LTP 570-2, March 4, 1963.

TABLE I - SUGGESTED DOCKING PERFORMANCE GOALS

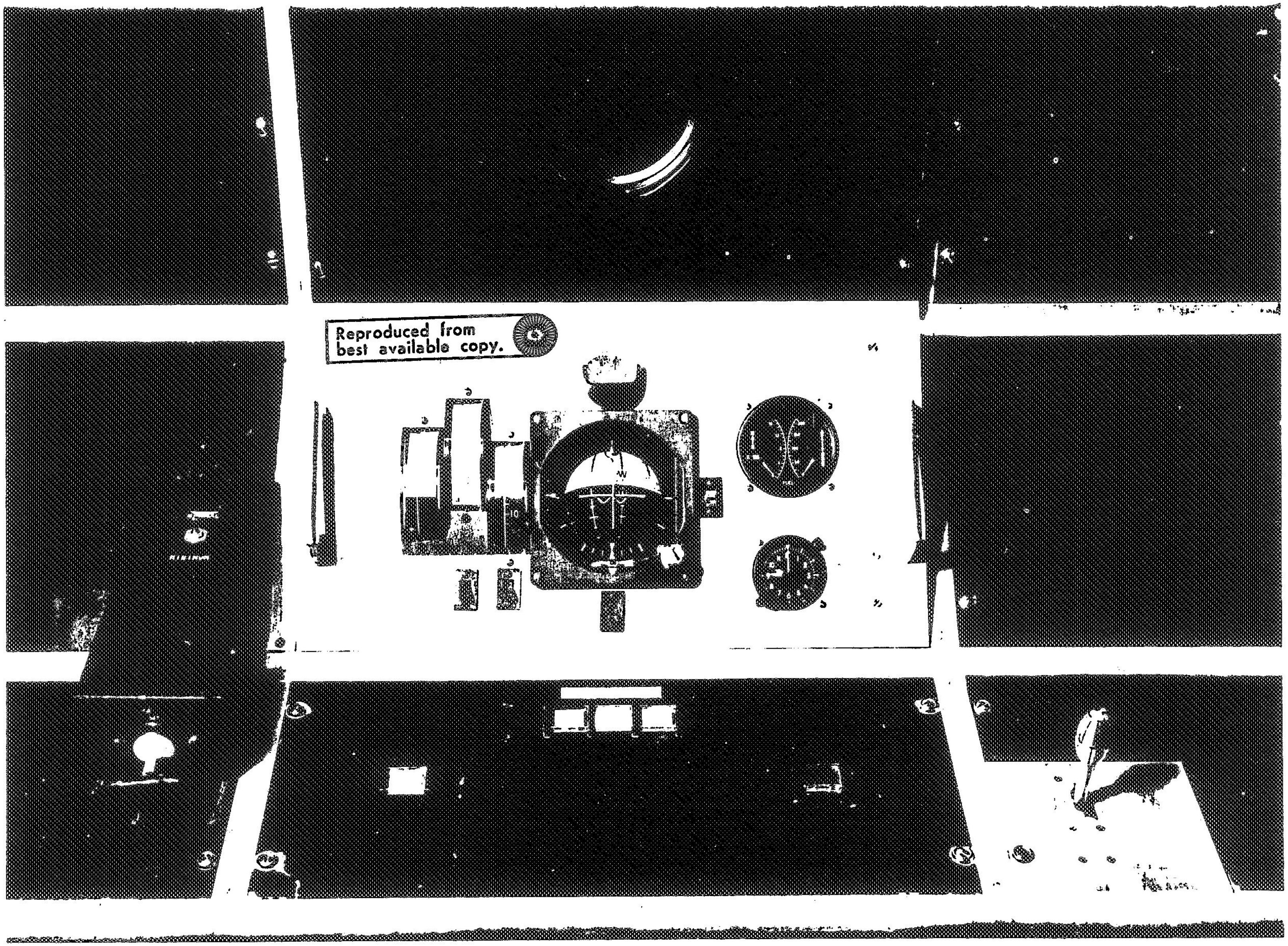
SYMBOL	DESCRIPTION	PERFORMANCE*
$\sqrt{y^2 + z^2}$	Radial displacement	$\frac{1}{2}$ ft
	Angle between docking planes	.087 rad (5 deg)
$\dot{x}$	Axial velocity	1 ft/sec
$\sqrt{\dot{y}^2 + \dot{z}^2}$	Lateral velocity	$\frac{1}{2}$ ft/sec
p,q,r	Body rotational rates	.017 rad/sec (1 deg/sec)

*The limiting contact rates, displacements, and attitudes required of the pilot should be more stringent than those dictated by structural requirements (in the interest of safety). The limiting contact conditions prescribed from structural considerations are, in the order taken above: 1 ft, .175 rad (10 deg), 2 ft/sec, 1 ft/sec, and .087 rad/sec (5 deg/sec).

	ADJECTIVE RATING	NUMERICAL RATING	DESCRIPTION	PRIMARY MISSION ACCOMPLISHED?
NORMAL OPERATION	SATISFACTORY	1	EXCELLENT, INCLUDES OPTIMUM	YES
		2	GOOD, PLEASANT TO FLY	YES
		3	SATISFACTORY, BUT WITH SOME MILDLY UNPLEASANT CHARACTERISTICS	YES
EMERGENCY OPERATION	UNSATISFACTORY	4	ACCEPTABLE, BUT WITH UNPLEASANT CHARACTERISTICS	YES
		5	UNACCEPTABLE FOR NORMAL OPERATION	DOUBTFUL
		6	ACCEPTABLE FOR EMERGENCY CONDITION ONLY*	DOUBTFUL
NO OPERATION	UNACCEPTABLE	7	UNACCEPTABLE EVEN FOR EMERGENCY CONDITION*	NO
		8	UNACCEPTABLE - DANGEROUS	NO
		9	UNACCEPTABLE - UNCONTROLLABLE	NO
	CATASTROPHIC	10	MOTIONS POSSIBLY VIOLENT ENOUGH TO PREVENT PILOT ESCAPE	

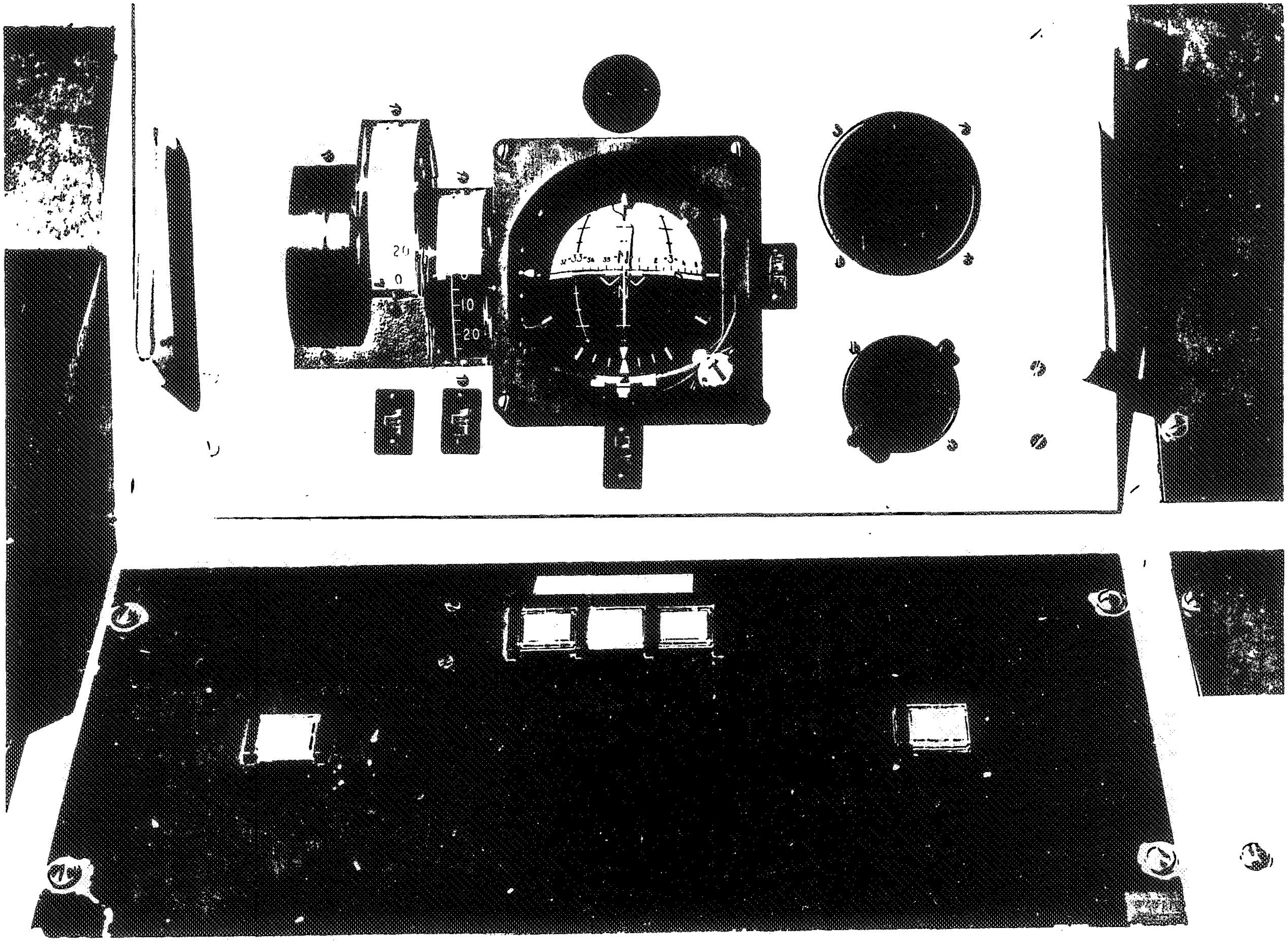
*(FAILURE OF A STABILITY AUGMENTER)

TABLE II - PILOT OPINION RATING SYSTEM



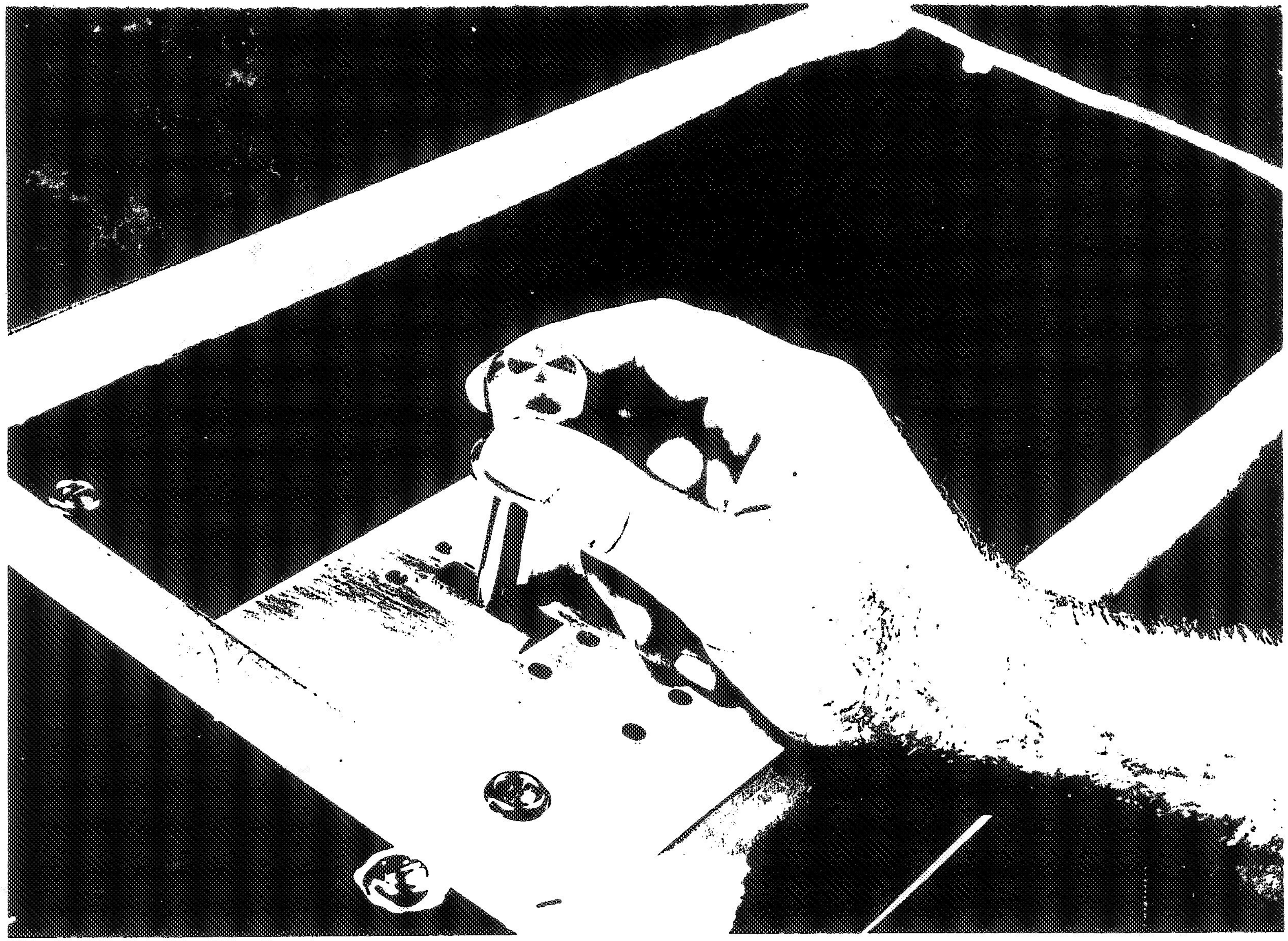
(a) Simulated Cockpit

Figure 1.- Photographs



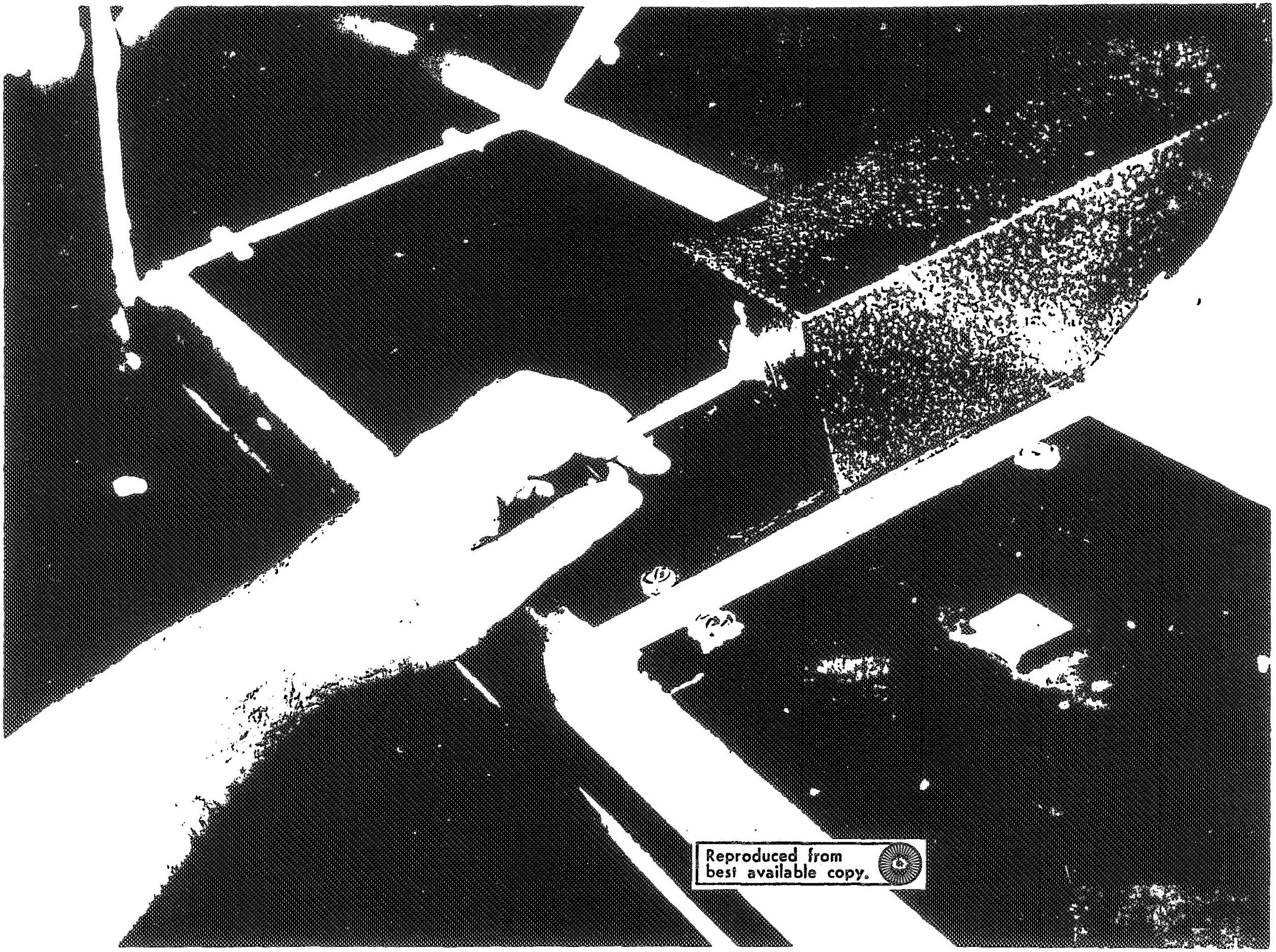
(b) Instrument Displays

Figure 1. - Continued



(c) Attitude Controller

Figure 1.- Continued



(d) Translation Controller

Figure 1 - Concluded

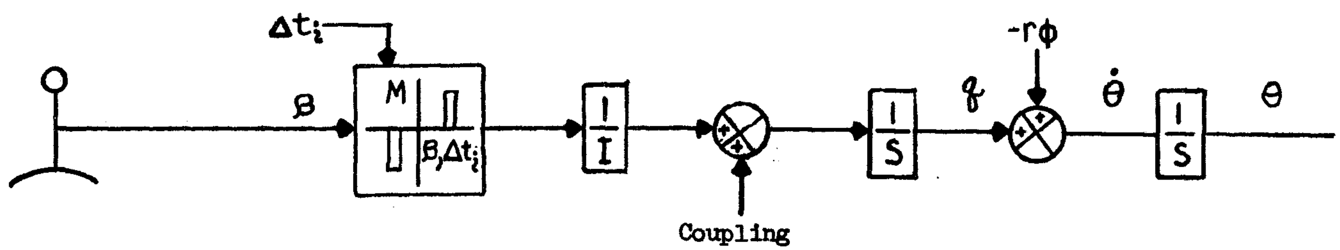


Figure 2.- Attitude Control Loop (Fixed-Impulse)

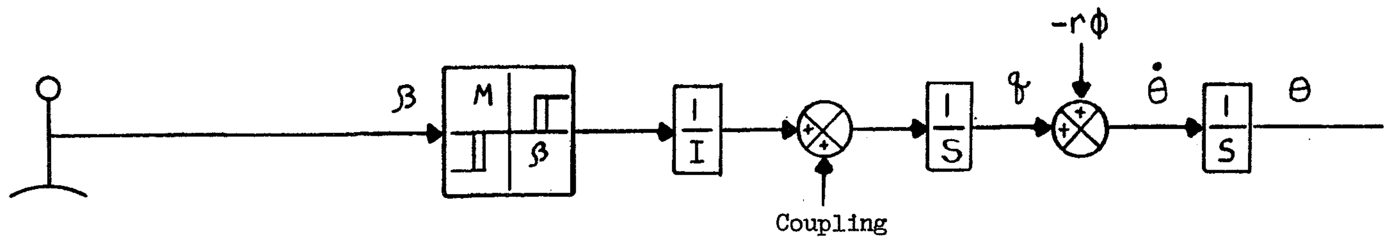


Figure 3.- Attitude Control Loop (ON-OFF)

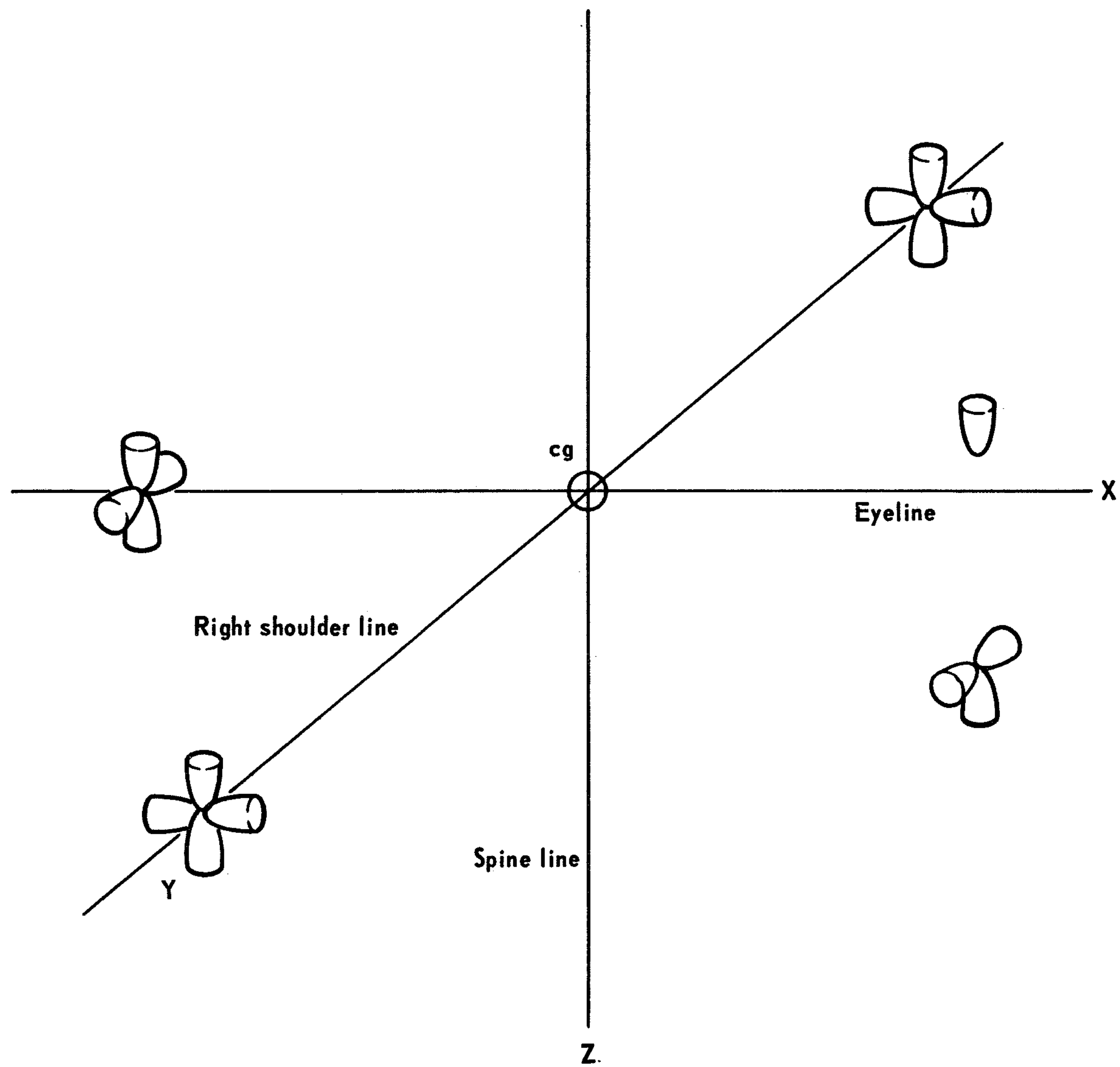


Figure 4.- LEM Thruster Arrangement (Light Configuration)

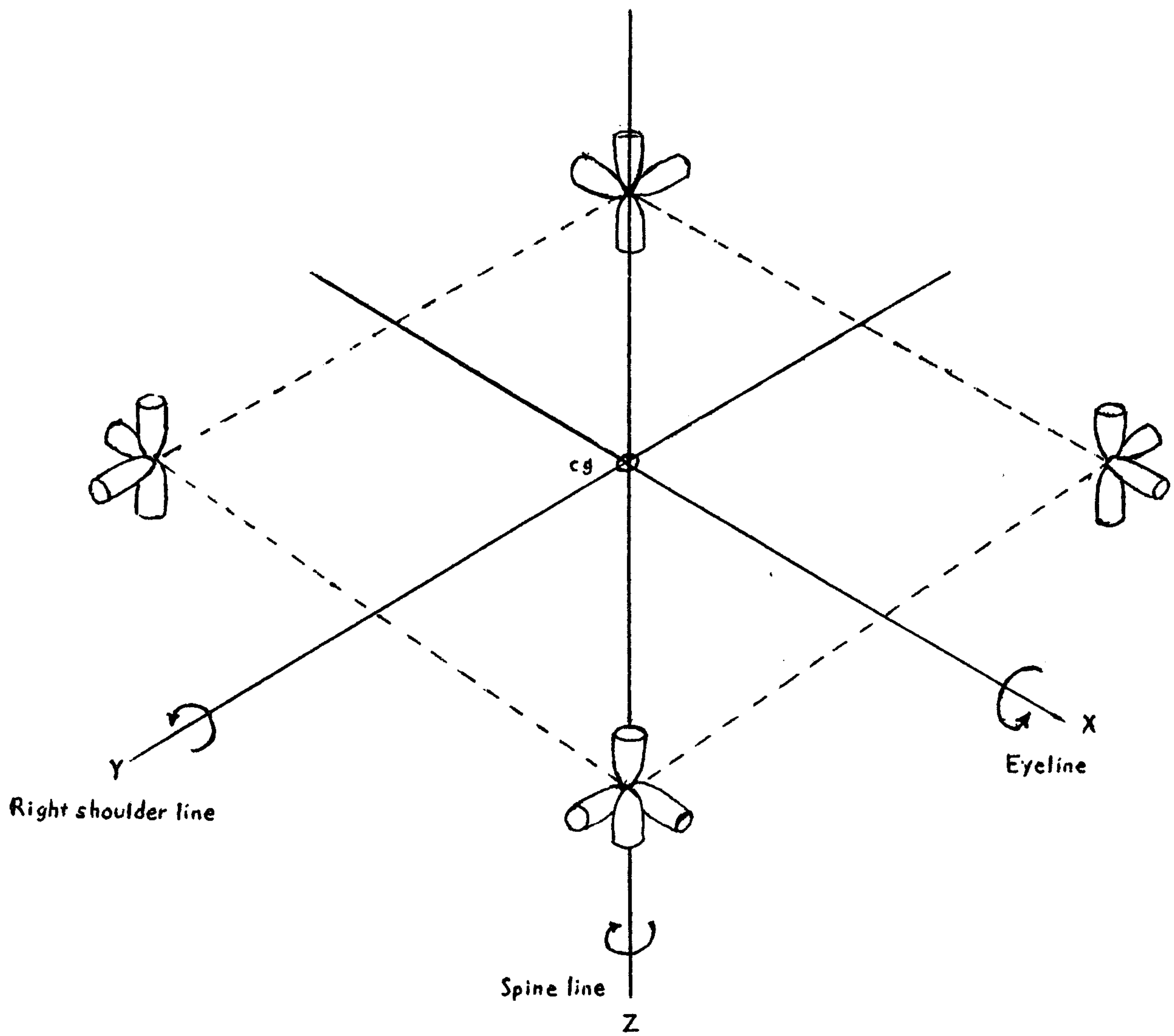
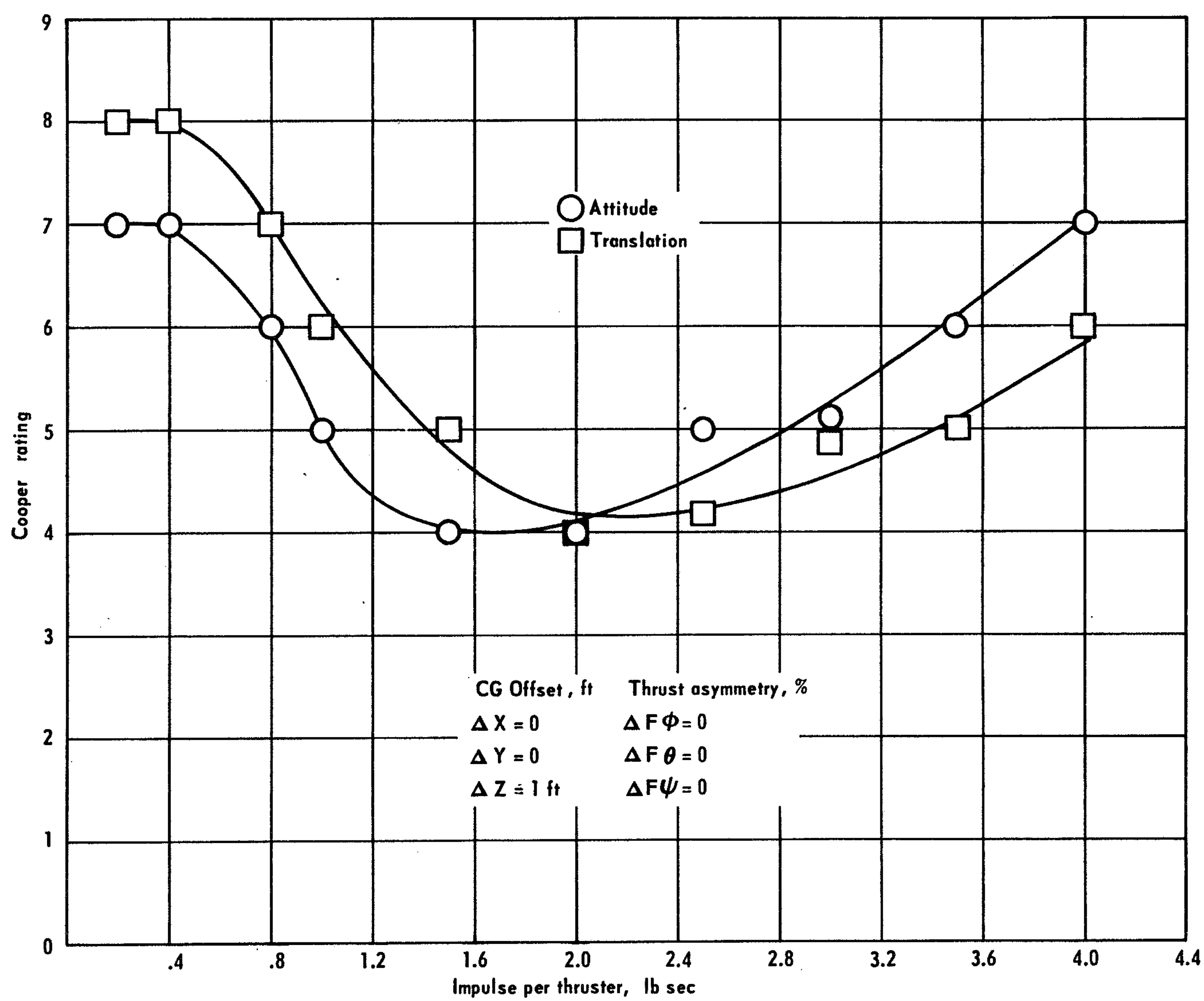
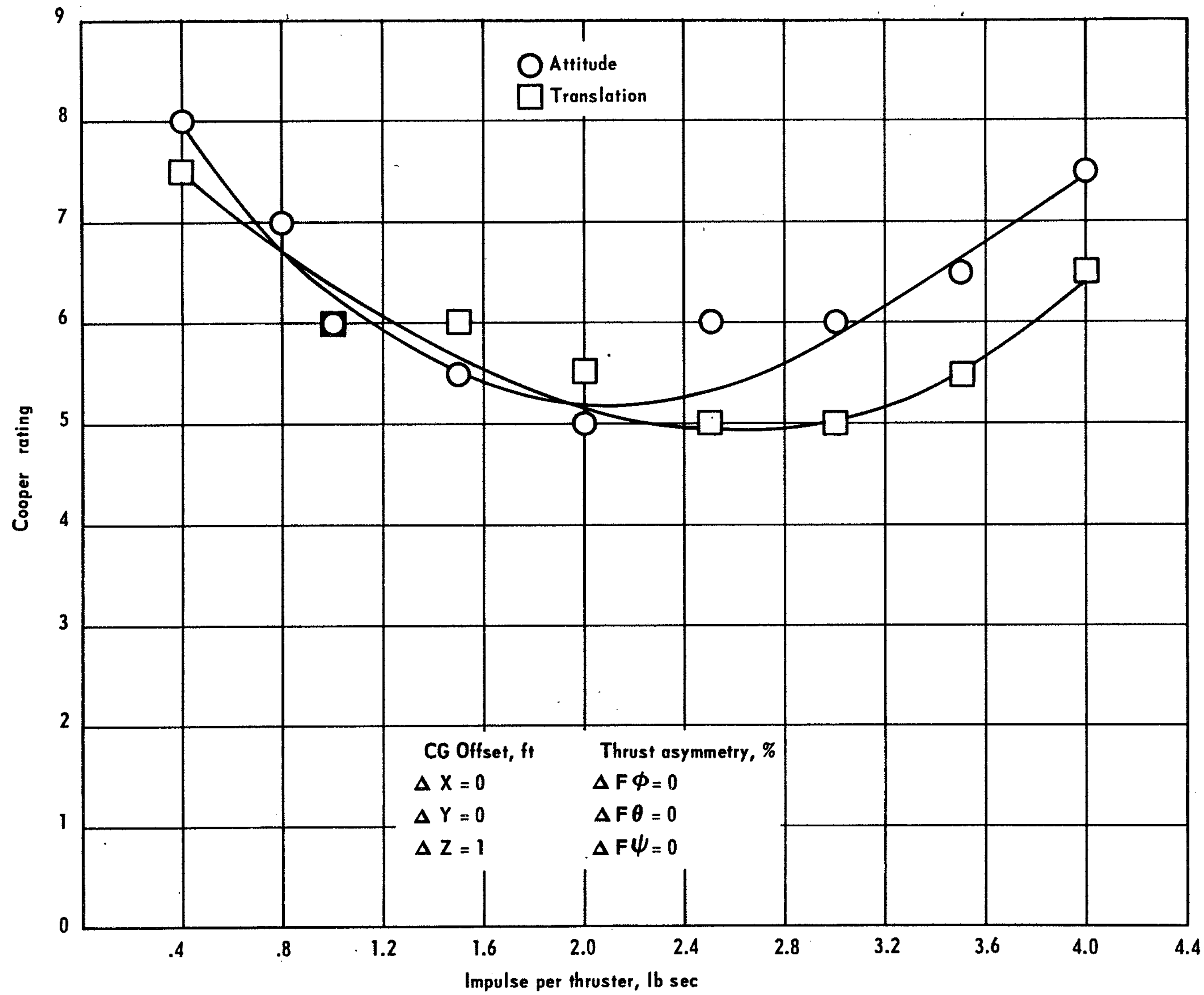


Figure 5.- LEM Thruster Arrangement (Heavy Configuration)



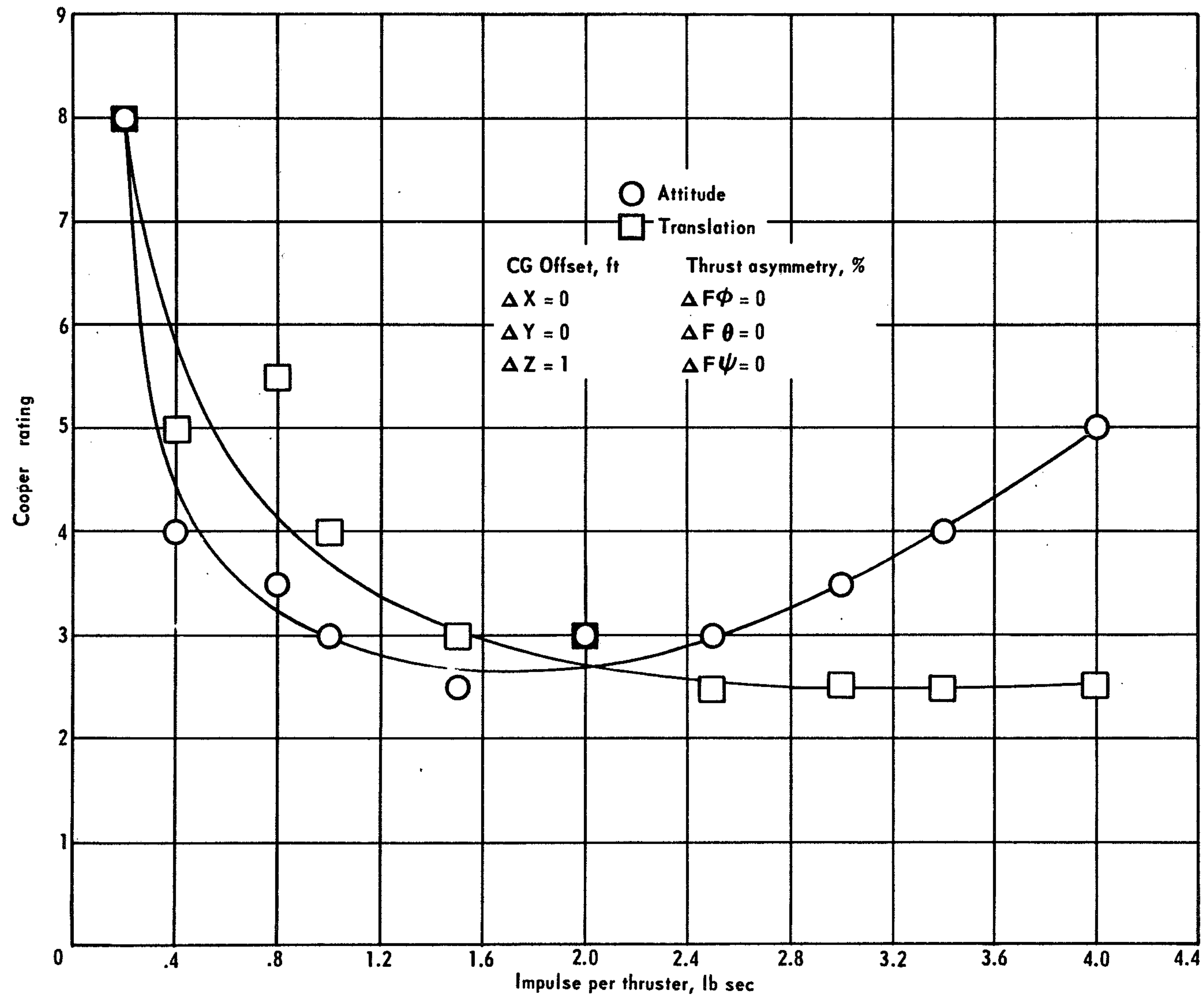
(a) Pilot A

Figure 6.- Pilot Rating of LEM Docking Control Quality as a Function of Thruster Impulse Value (Light Configuration)



(b) Pilot B

Figure 6.- Continued



(c) Pilot C

Figure 6.- Concluded

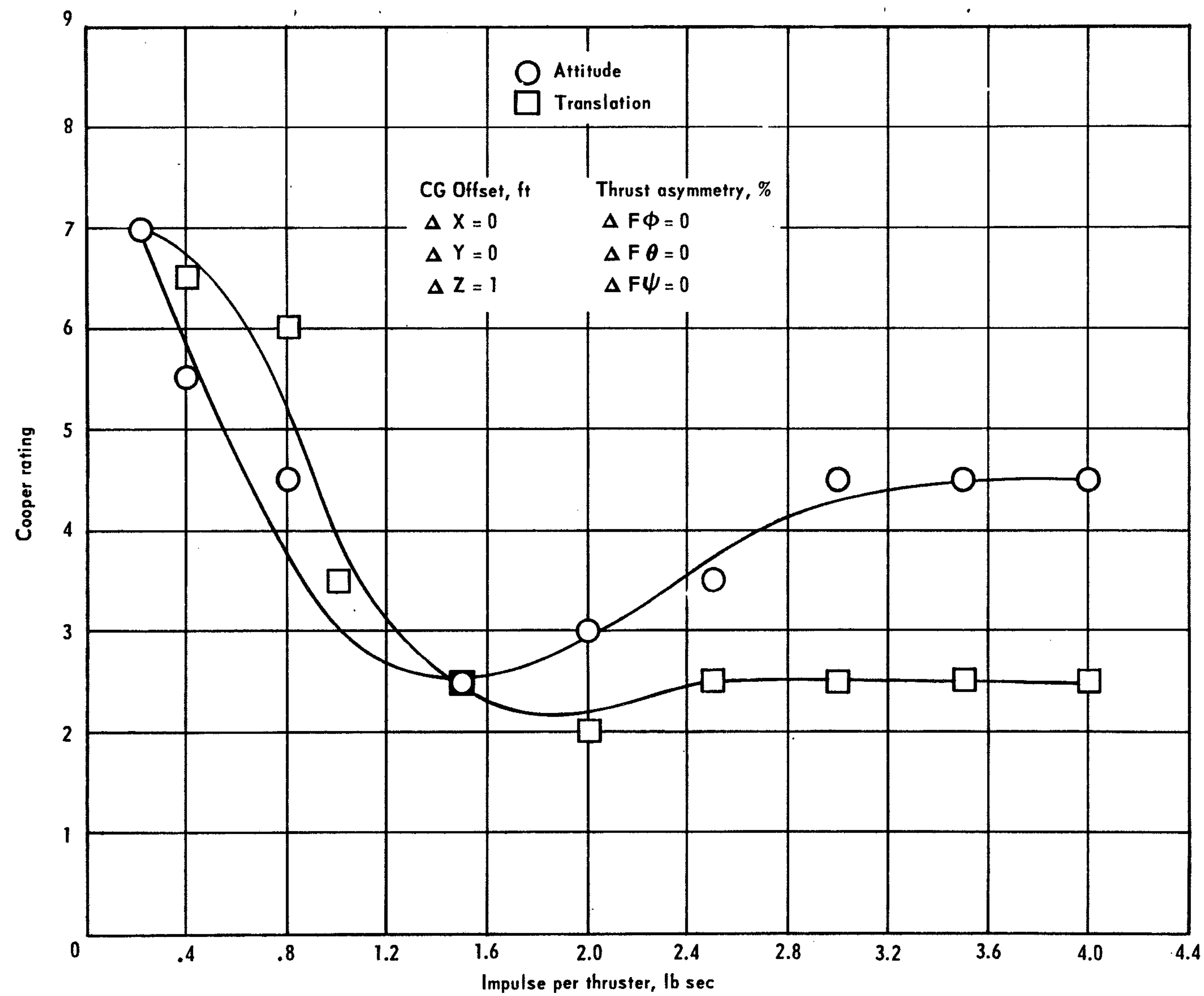
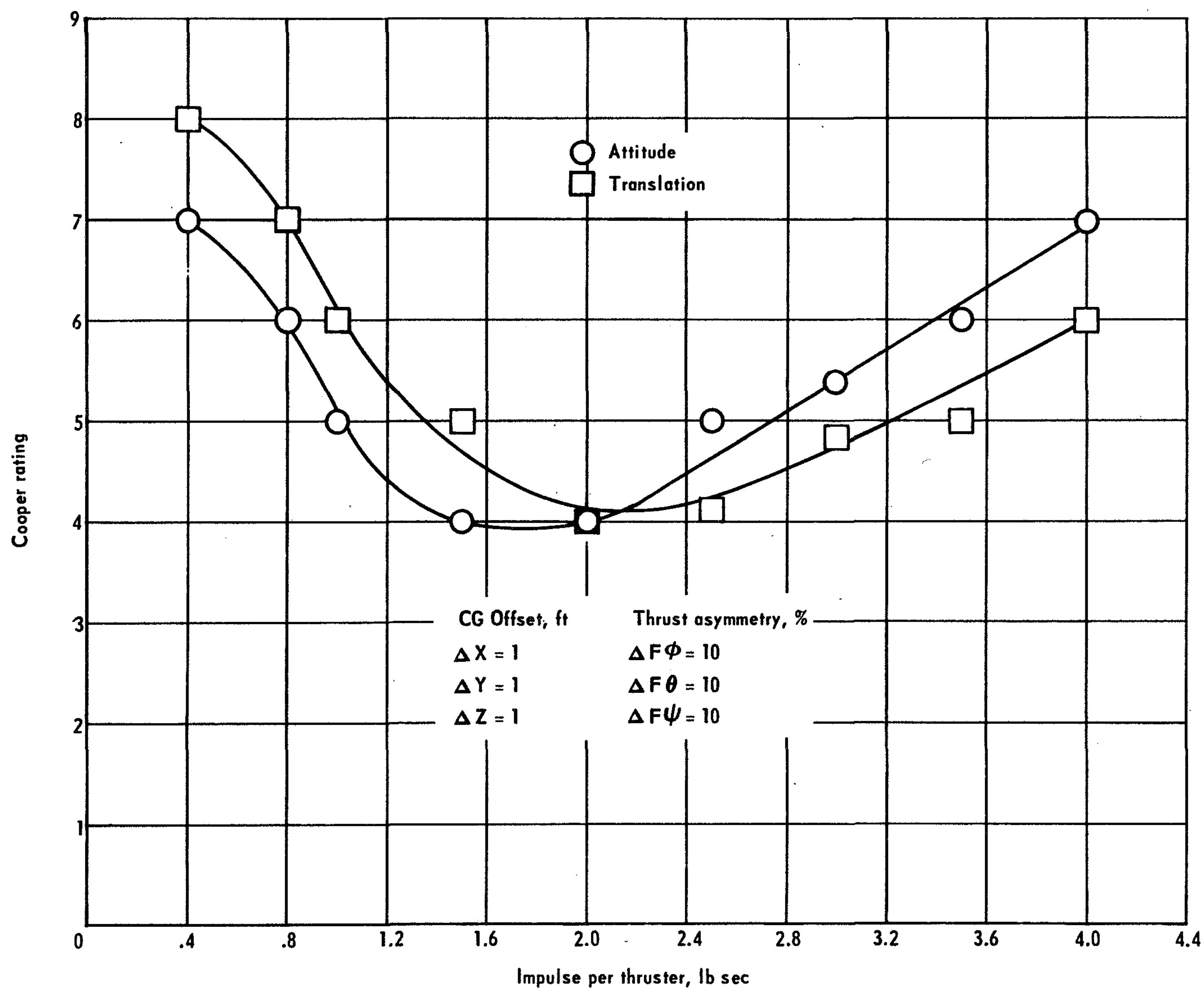
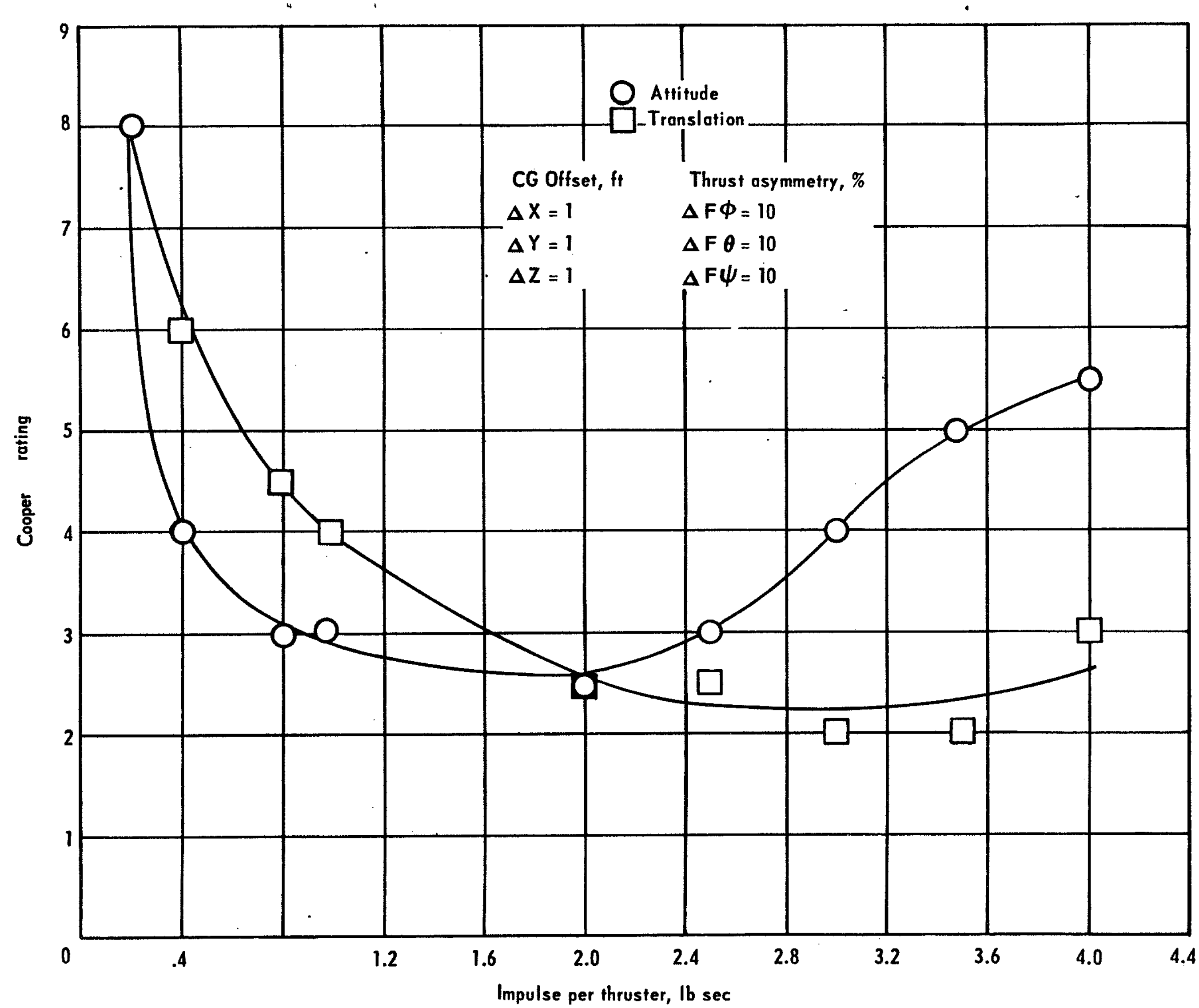


Figure 7.- Pilot Rating of LEM Docking Control Quality as a Function of Thruster Impulse Value (Heavy Configuration)



(a) Pilot A.

Figure 8.- Pilot Rating of LEM Docking Control Quality as a Function of Thruster Impulse Value (Light Configuration)



(b) Pilot C

Figure 8.- Concluded

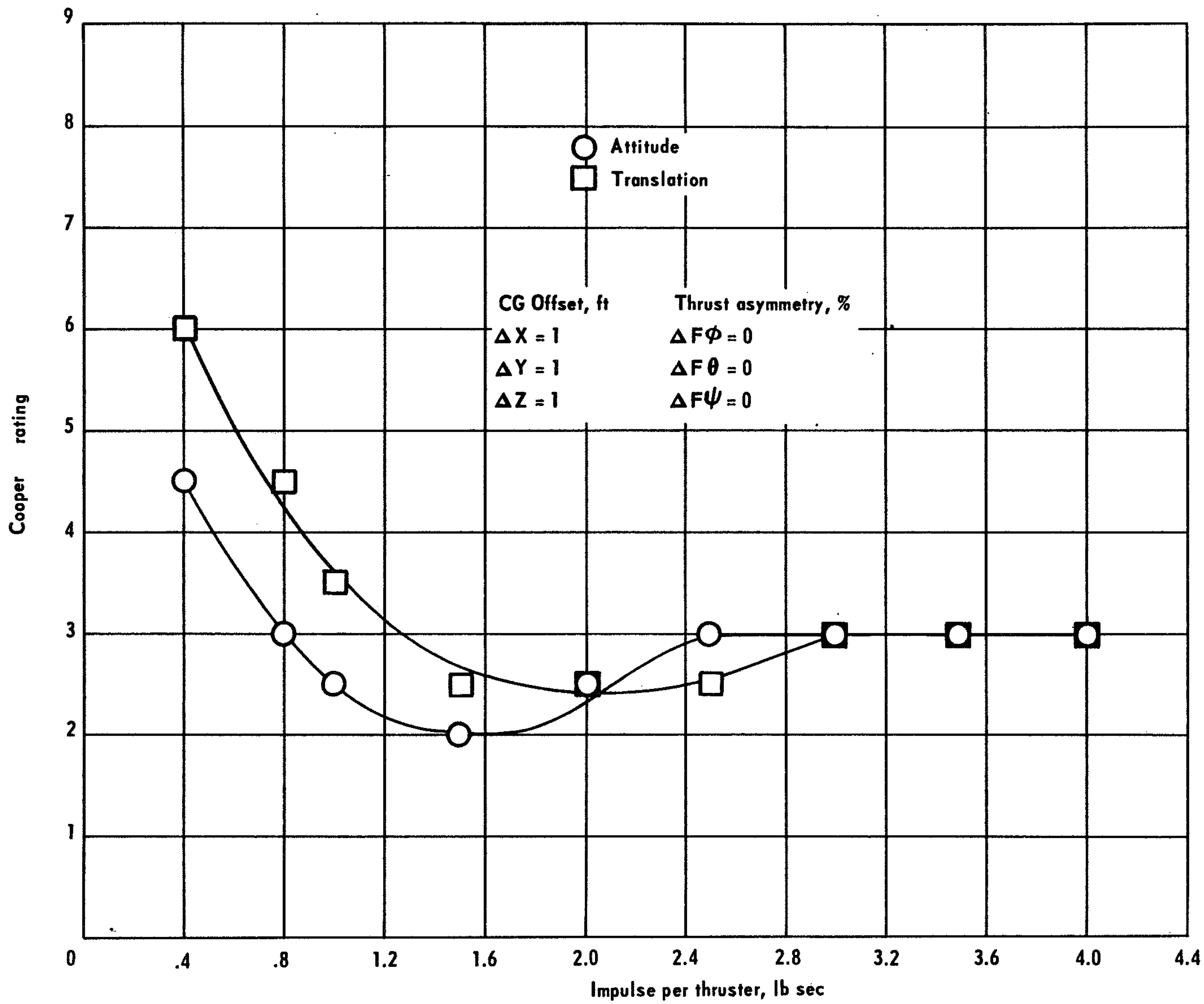
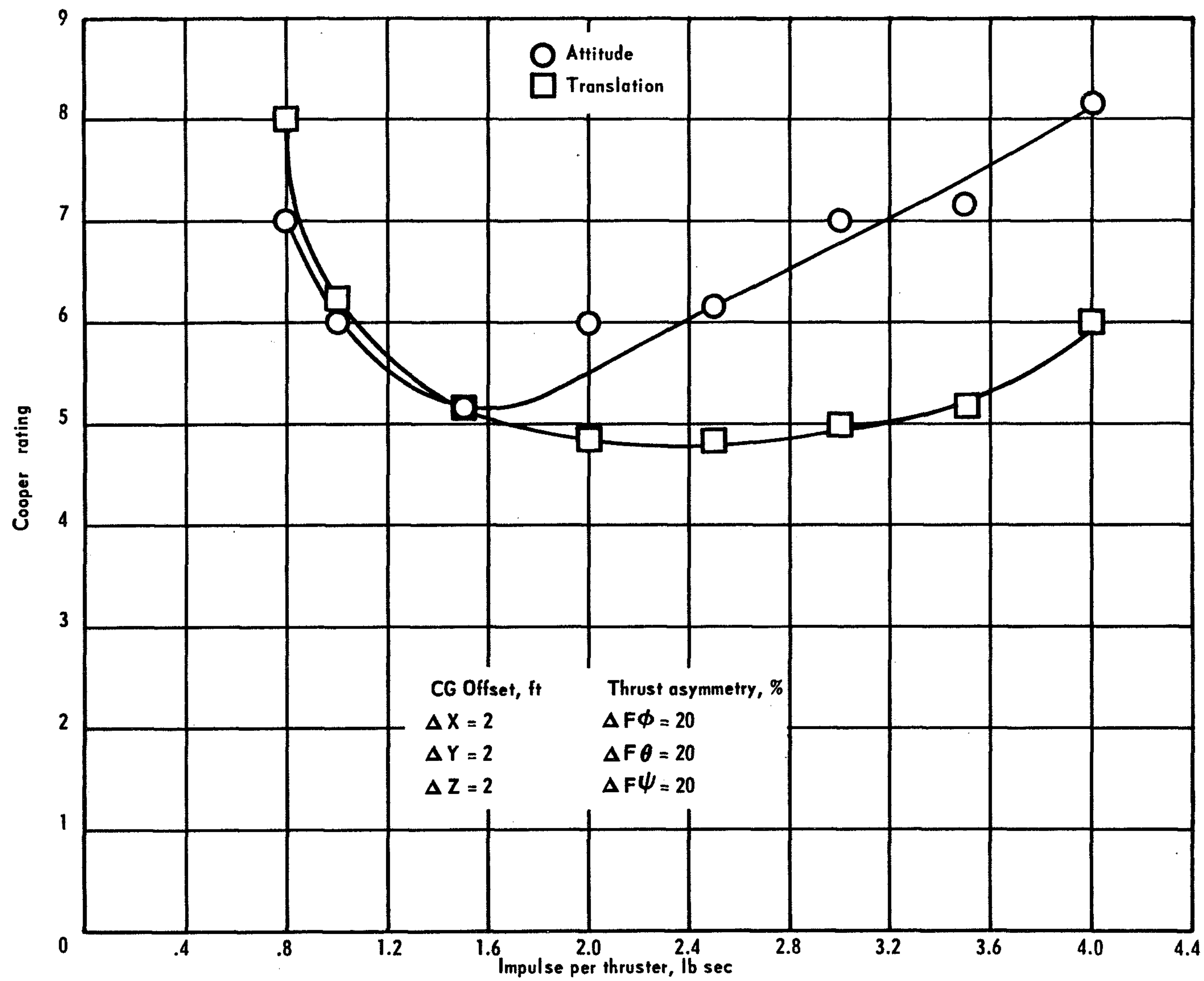
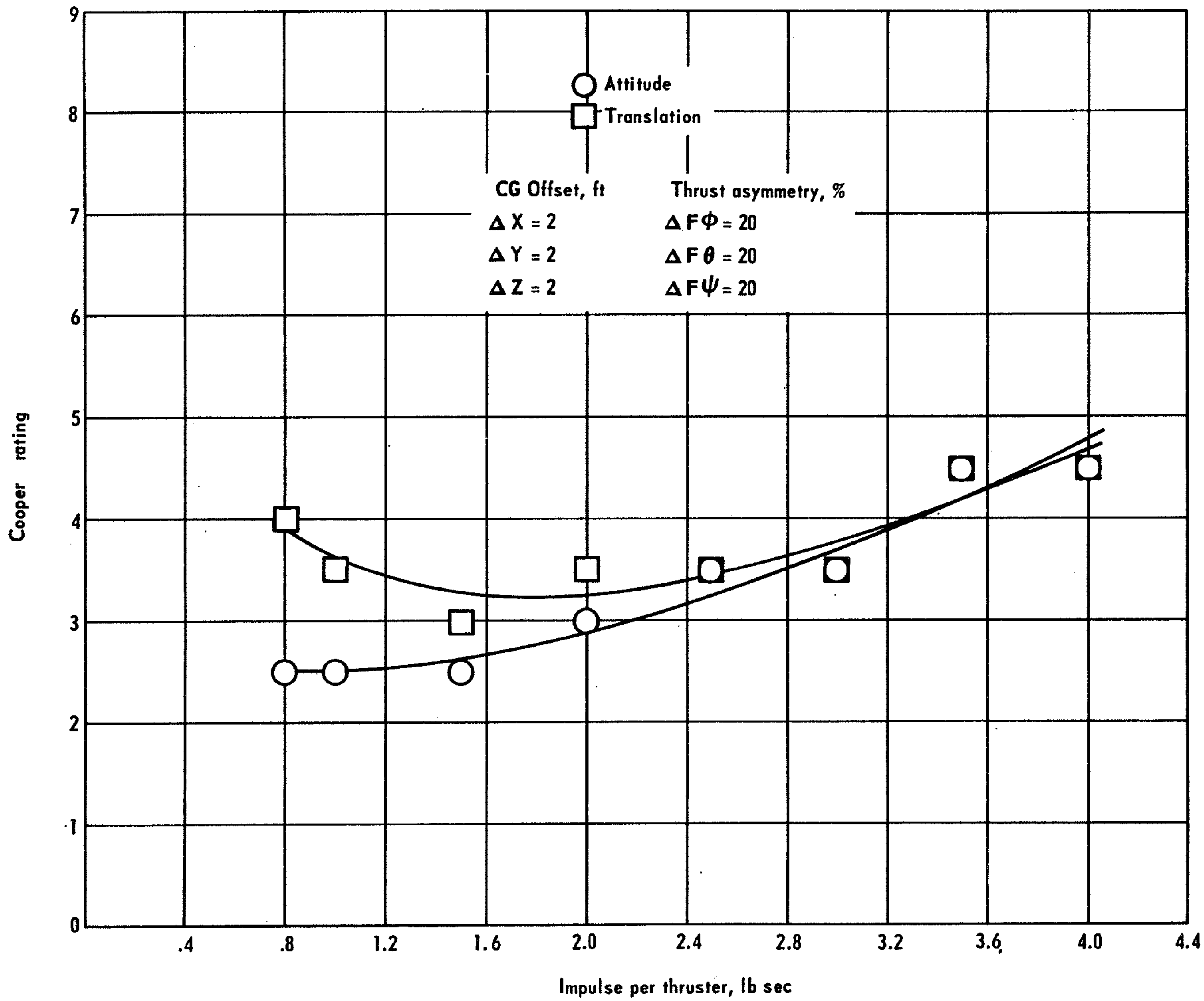


Figure 9.- Pilot Rating of LEM Docking Control Quality as a Function of Thruster Impulse Value (Heavy Configuration)



(a) Pilot A

Figure 10.- Pilot Rating of LEM Docking Control Quality as a Function of Thruster Impulse Value (Light Configuration)



(b) Pilot C

Figure 10.- Concluded

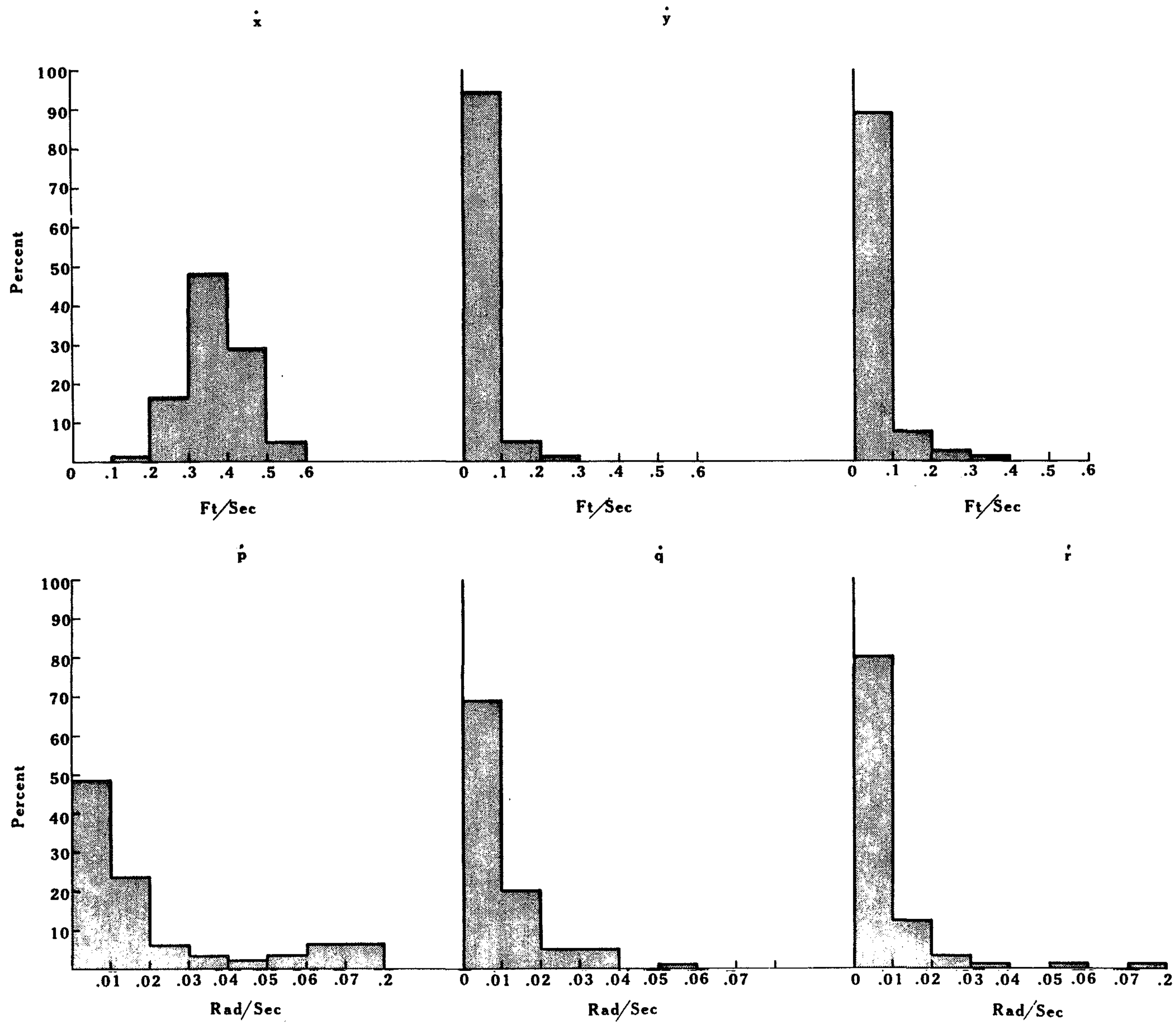


Figure 11.- Distribution of LEM Angular and Translational Rates at

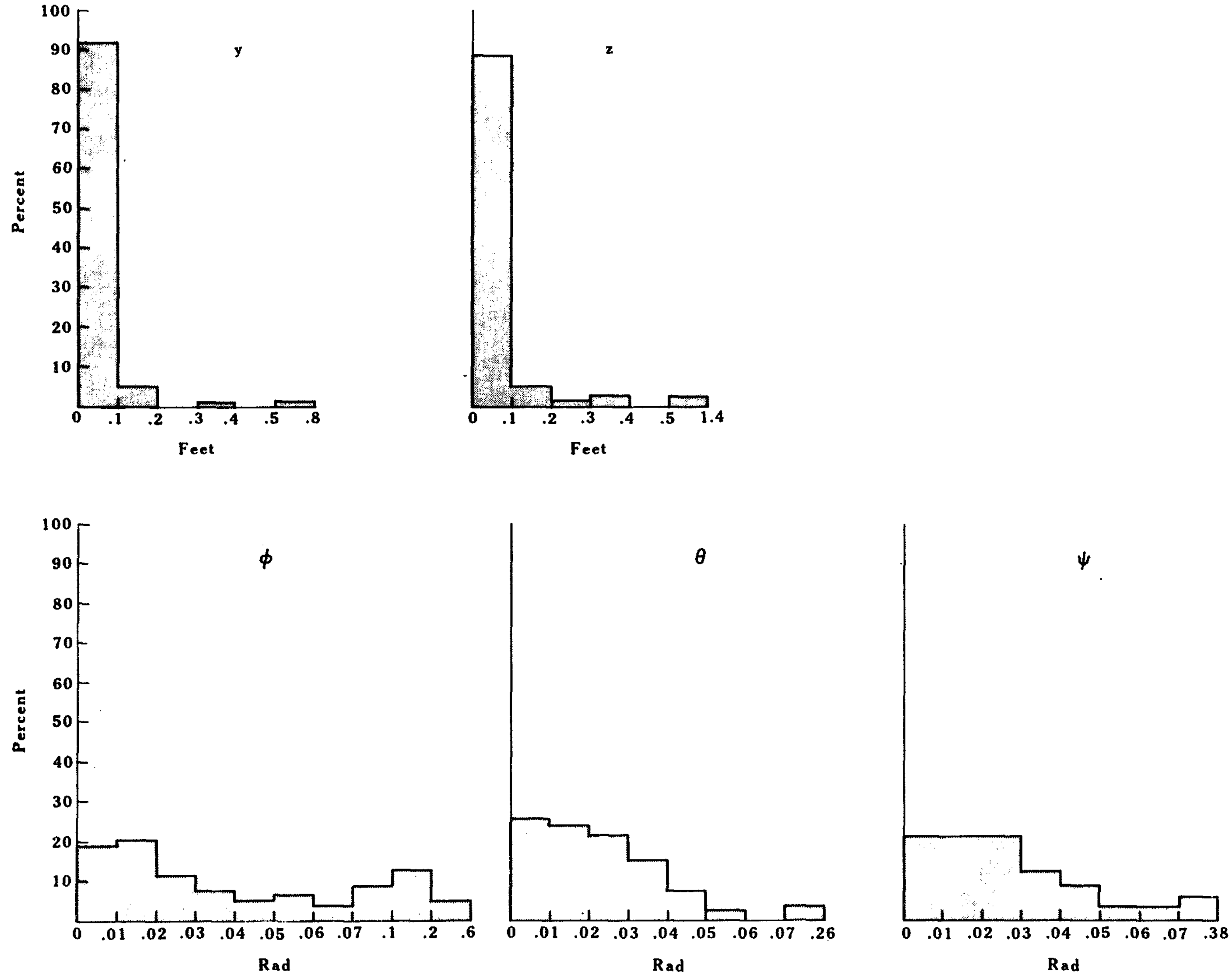


Figure 12.- Distribution of LEM Angular and Translational Displacements at Docking Contact

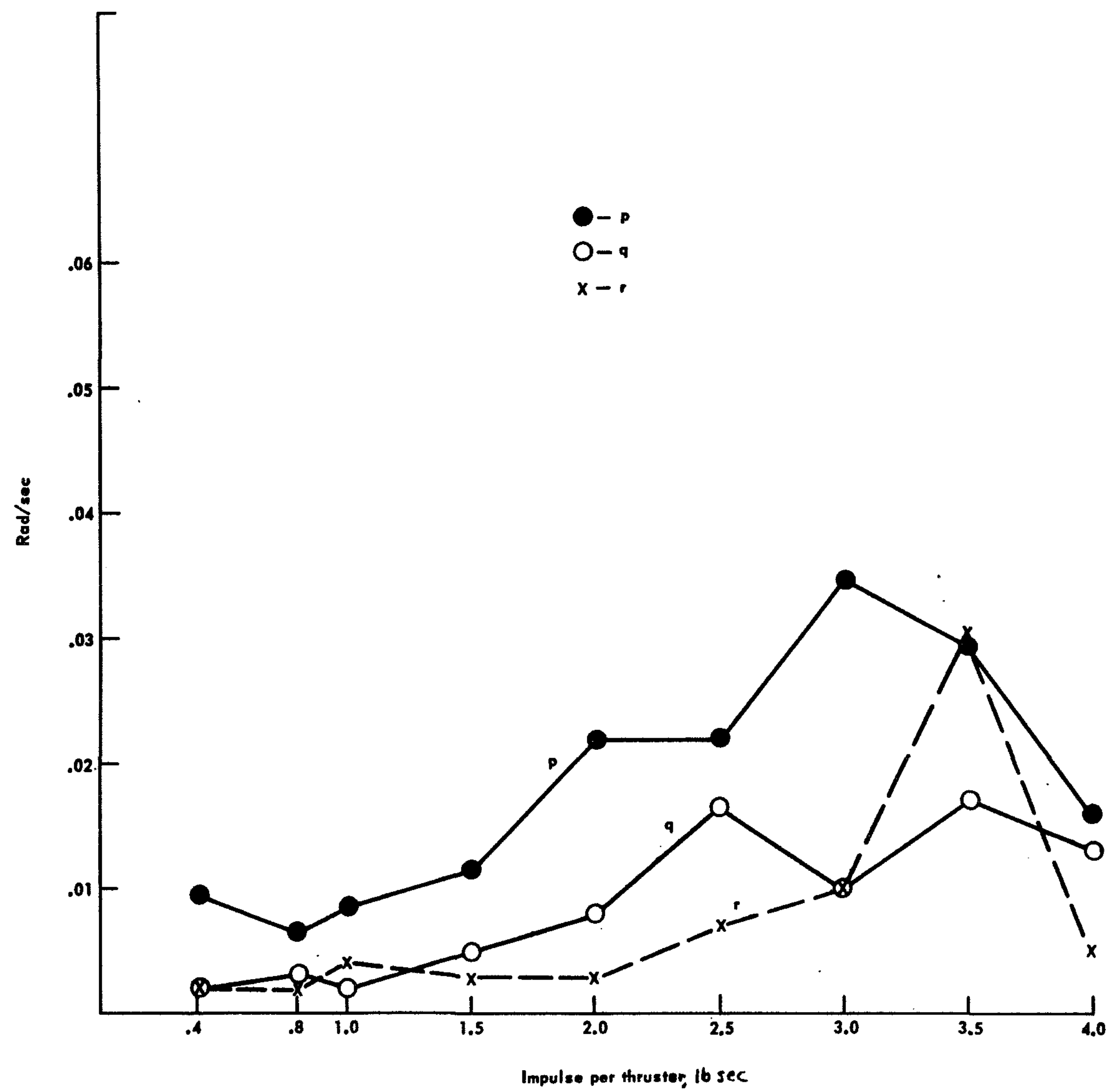


Figure 13.- LEM Angular Rate at Dock as a Function of Impulse Value; Average of all Runs

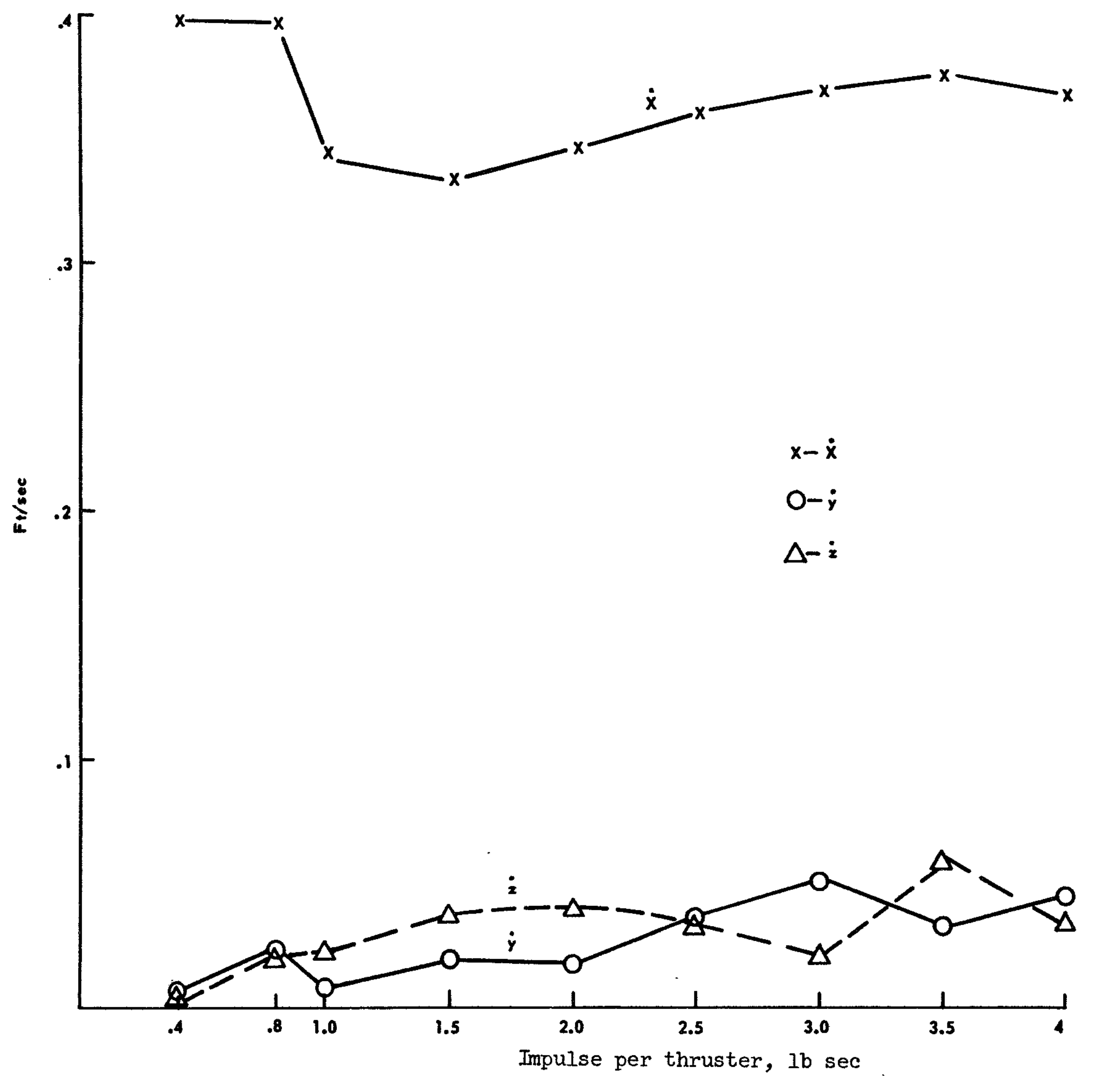


Figure 14.- LEM Translational Rate at Dock as a Function of Impulse Value; Average of all Runs

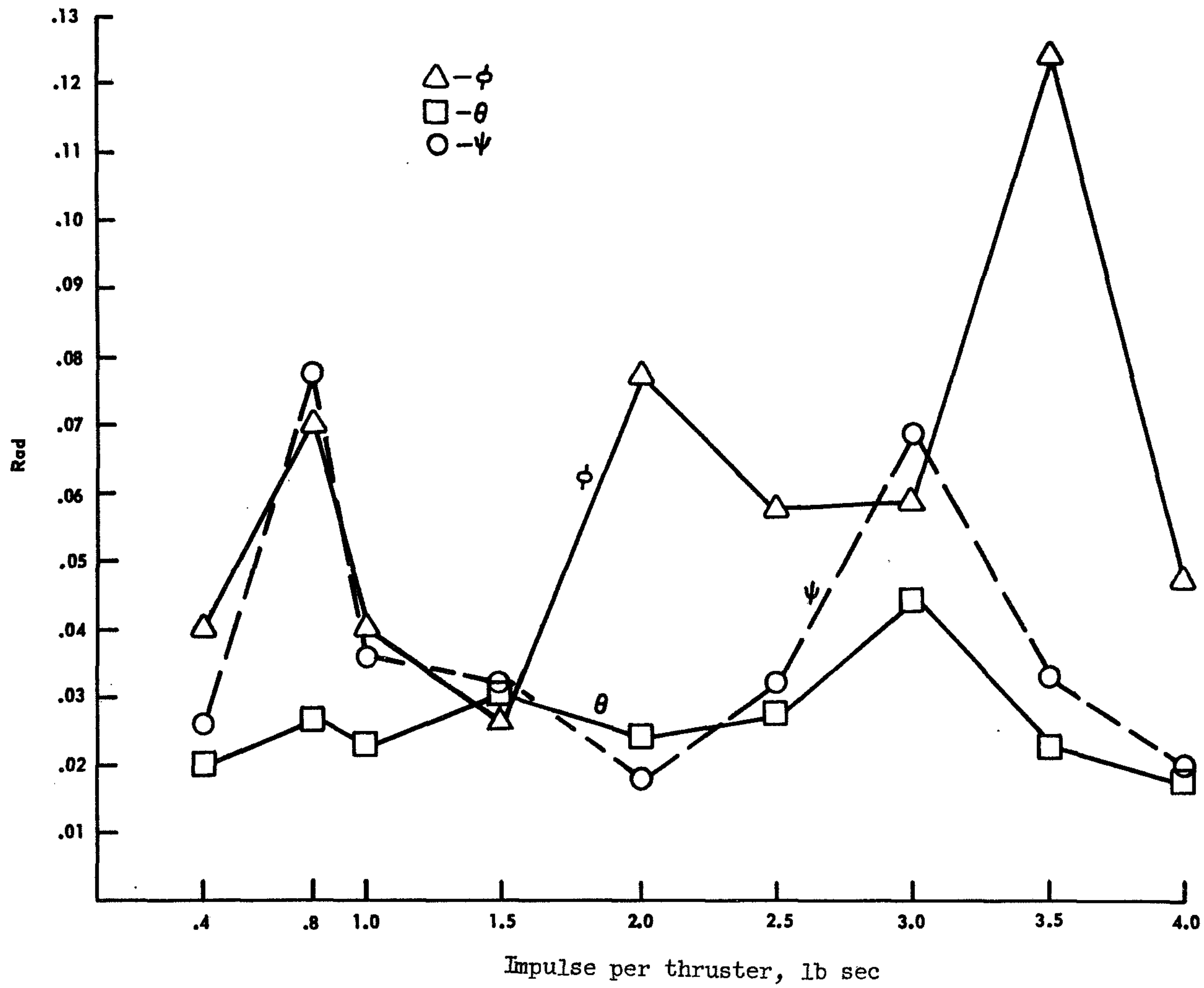


Figure 15.- LEM Attitude at Dock as a Function of Impulse Value; Average of all Runs

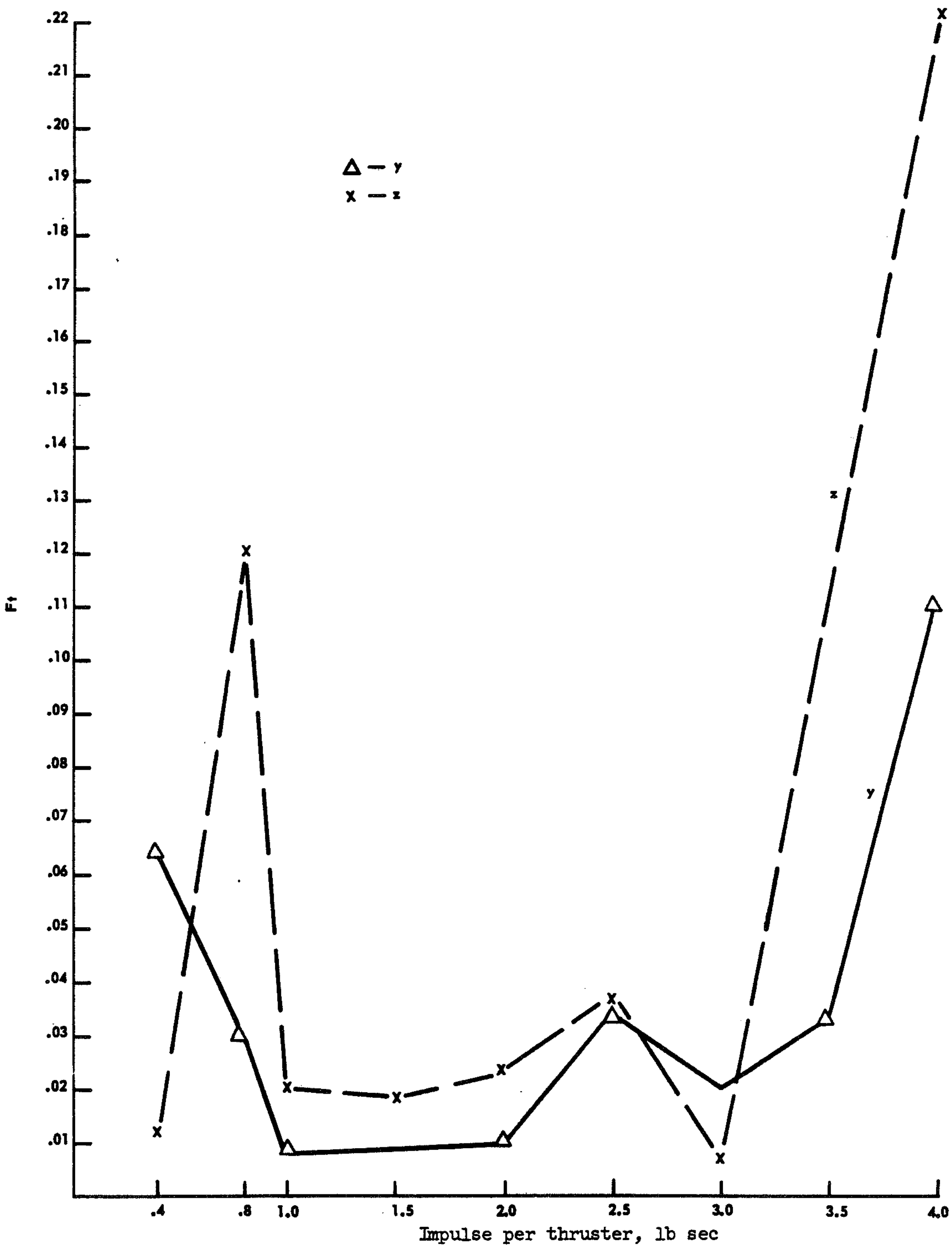


Figure 16.- LEM Lateral Displacement at Dock as a Function of Impulse Value; Average of all Runs

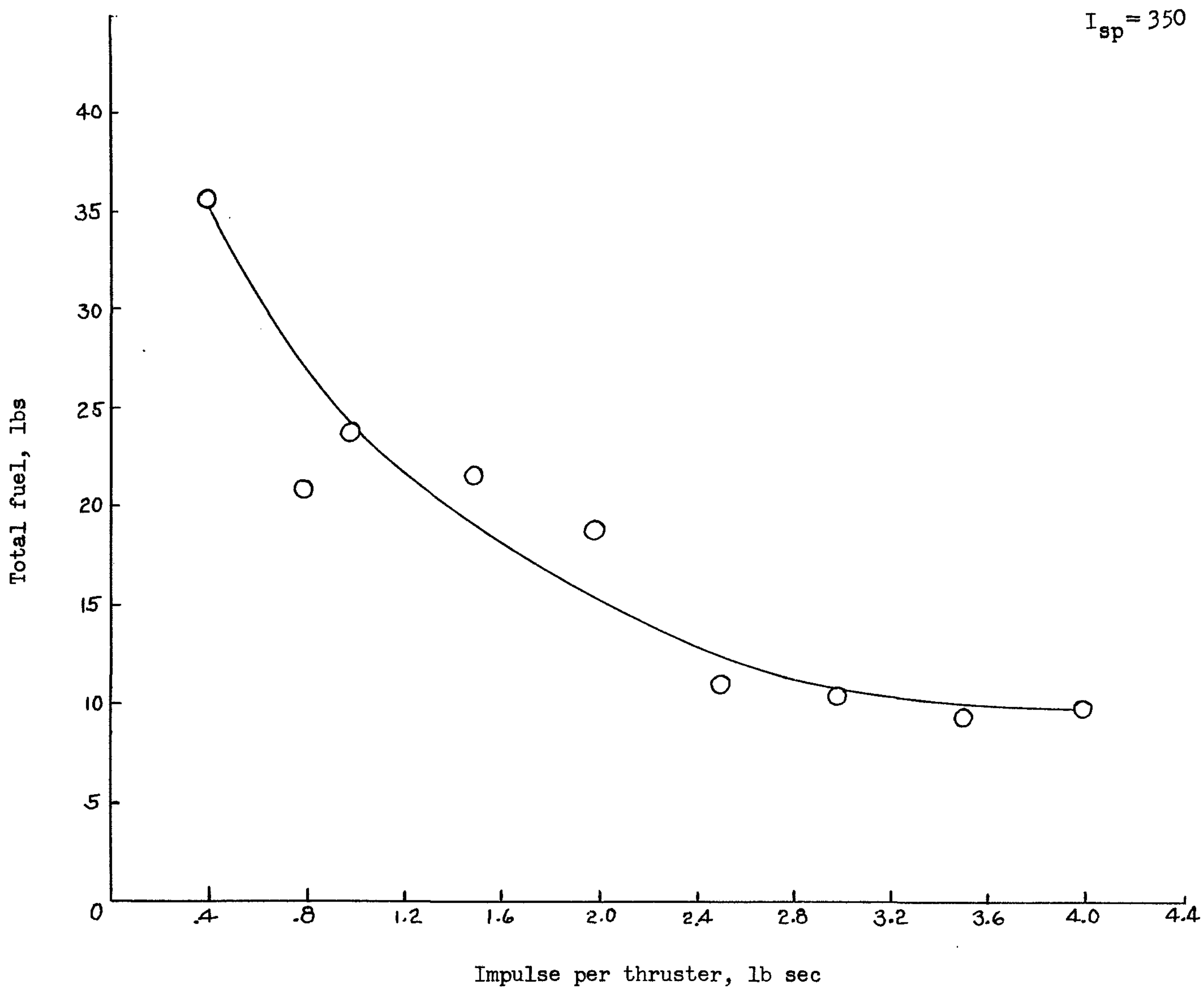


Figure 17.- Total Fuel Used in Docking as a Function of Impulse Per Thruster; Average of all Runs

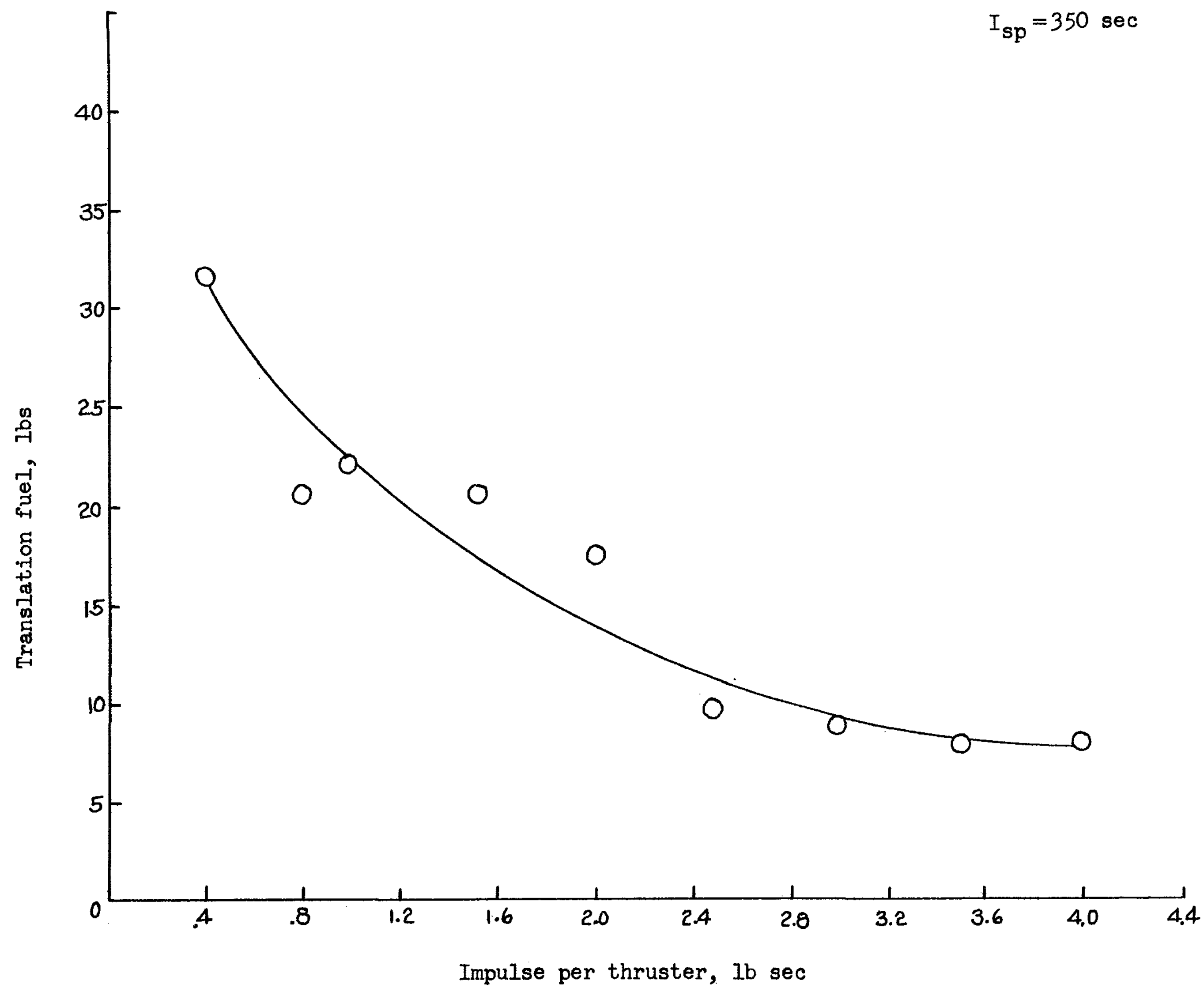


Figure 18.- Translation Fuel Used in Docking as a Function of Impulse Per Thruster; Average of all Runs

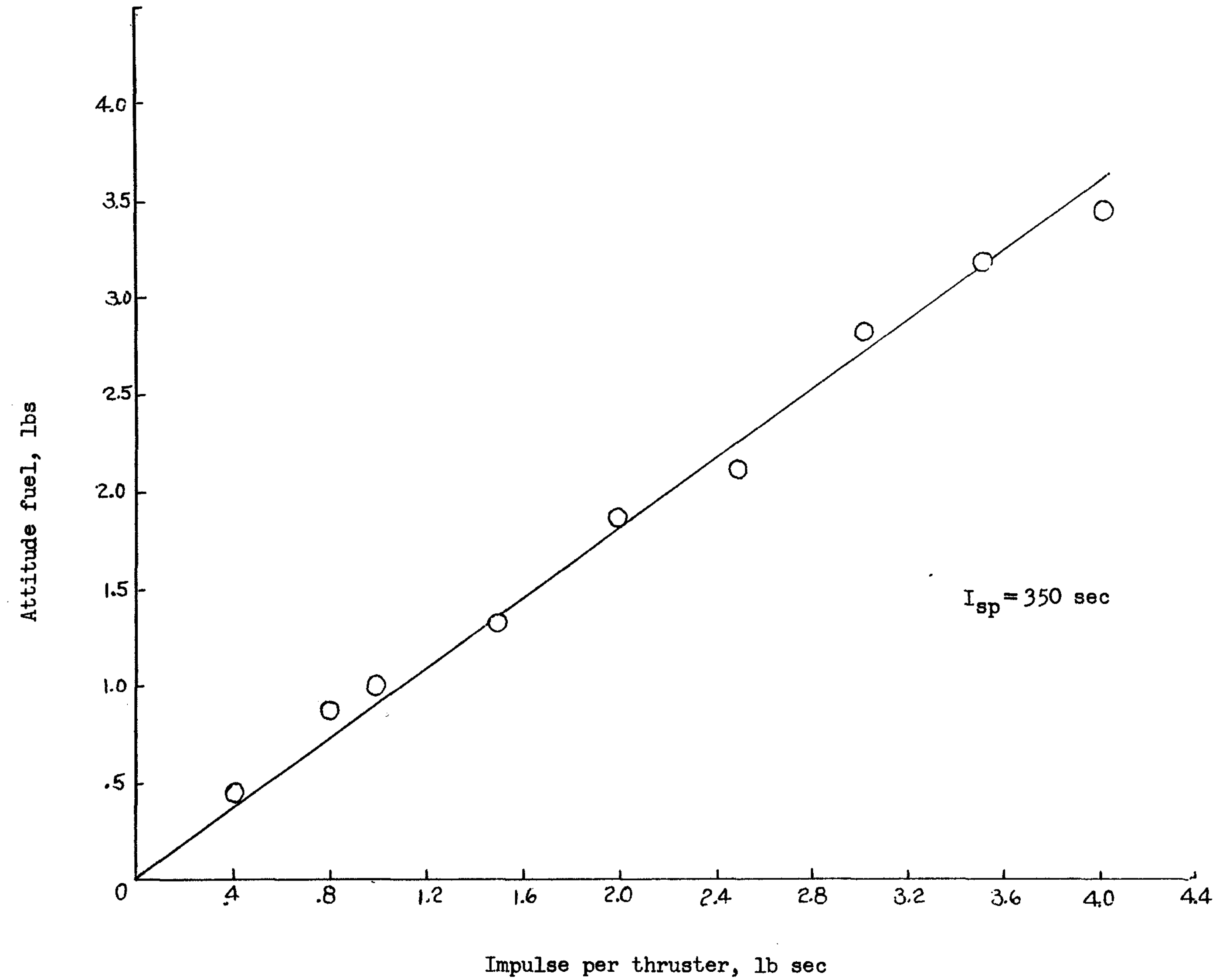


Figure 19.- Attitude Fuel Used in Docking as a Function of Impulse Per Thruster; Average of all Runs

APPENDIX

DOCKING EQUATIONS OF MOTION

1. LEM angular accelerations: (Neglecting products of inertia)

$$\dot{p} = \frac{L_c + F_y\Delta z - F_z\Delta y + (I_{yy} - I_{zz})qr}{I_{xx}}$$

$$\dot{q} = \frac{M_c + F_z\Delta x - F_x\Delta z - (I_{xx} - I_{zz})pr}{I_{yy}}$$

$$\dot{r} = \frac{N_c + F_x\Delta y - F_y\Delta x - (I_{yy} - I_{xx})pq}{I_{zz}}$$

2. Euler kinematic relations: (Assuming small angles)

$$\dot{\phi} = p + r\theta \cong p + \dot{\psi}\sin\theta$$

$$\dot{\theta} = q - r\phi \cong q\cos\phi - r\sin\phi$$

$$\dot{\psi} = r + q\phi \cong \frac{q\sin\phi + r\cos\phi}{\cos\theta}$$

3. LEM translational accelerations with respect to the CM:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} F_x + \Delta F_\theta + \Delta F_\psi \\ F_y \\ F_z + \Delta F_\phi \end{bmatrix},$$

where

$$A = \begin{bmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\phi \\ -\theta & \phi & 1 \end{bmatrix} \cong \begin{bmatrix} c\psi\, c\theta & c\psi\, s\theta s\phi - s\psi\, c\phi & c\psi s\theta c\phi + s\psi s\phi \\ s\psi\, c\theta & s\psi\, s\theta s\phi + c\psi\, c\phi & s\psi s\theta c\phi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}$$

assuming small angles.

then

$$\ddot{x} = \frac{1}{m}\left[F_x + \Delta F_\theta + \Delta F_\psi - F_y\psi + (F_z + \Delta F_\phi)\theta\right]$$

$$\ddot{y} = \frac{1}{m}\left[(F_x + \Delta F_\theta + \Delta F_\psi) + F_y - (F_z + \Delta F_\phi)\phi\right]$$

$$\ddot{z} = \frac{1}{m}\left[-(F_x + \Delta F_\theta + \Delta F_\psi)\theta + F_y\phi + F_z + \Delta F_\phi\right].$$

4. Apparent translational velocity of the CM wrt the LEM:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} A \end{bmatrix}^{-1} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} - \underline{W} \times \underline{R} \quad \text{where } \underline{W} = ip + jq + kr$$

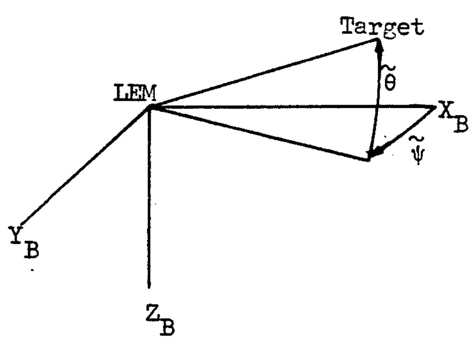
$$\underline{R} = ix_b + jy_b + kz_b$$

5. The Azimuth, Elevation, and Range of the CM wrt the LEM: (These are the spherical coordinates of the CM with respect to the LEM and describe the motions of the CM relative to the pilot of the LEM.)

$$\dot{R} = (u \cos\tilde{\psi} + v \sin\tilde{\psi})\cos\tilde{\theta} - w \sin\tilde{\theta}$$

$$R\dot{\tilde{\psi}}\cos\theta = -u \sin\tilde{\psi} + v \cos\tilde{\psi}$$

$$-R\dot{\tilde{\theta}} = (u \cos\tilde{\psi} + v \sin\tilde{\psi}) \sin\tilde{\theta} + w \cos\tilde{\theta}$$



$$R = (X_B^2 + Y_B^2 + Z_B^2)^{\frac{1}{2}}$$

$$\tilde{\psi} = \tan^{-1}\frac{X_B}{Y_B}$$

$$\tilde{\theta} = \tan^{-1}\frac{Z_B}{(X_B^2 + Y_B^2)^{\frac{1}{2}}}$$

where u,v,w are translational rates of the target wrt X_B, Y_B, Z_B.

6. Fuel weights:

$$W_{F_R} = \frac{1}{I_{sp}} \int \left(|F_\phi| + |F_\theta| + |F_\psi| \right) dt \qquad \text{in rotation}$$

$$W_{F_T} = \frac{I}{I_{sp}} \int \left(|F_x| + |F_y| + |F_z| \right) dt \qquad \text{in translation}$$